Theory of physical structures:

hidden symmetries in some basic laws of general physics

Alexey Nenashev

Institute of Semiconductor Physics

Novosibirsk, Russia

Theory of Physical Structures

is an unusual point of view on

“usual” laws of physics and

geometry, such as

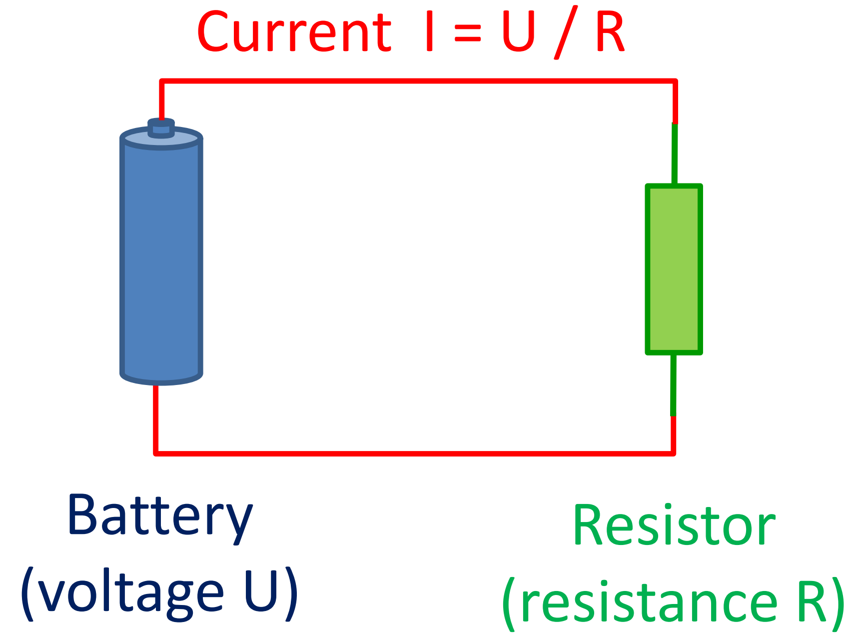
- Newton’s second law: $F = ma$,
- Ohm’s law: $U = IR$,
- distance between two points:

  $r_{ab}^2 = (x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2$,

etc.

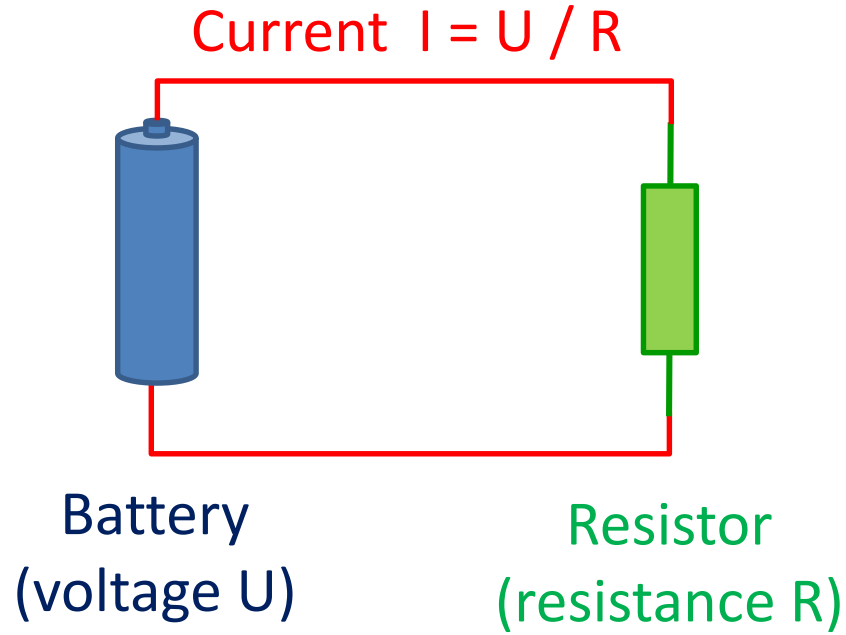
Yuri Kulakov
suggested this theory in 1960s

Ohm's law



Ohm's law

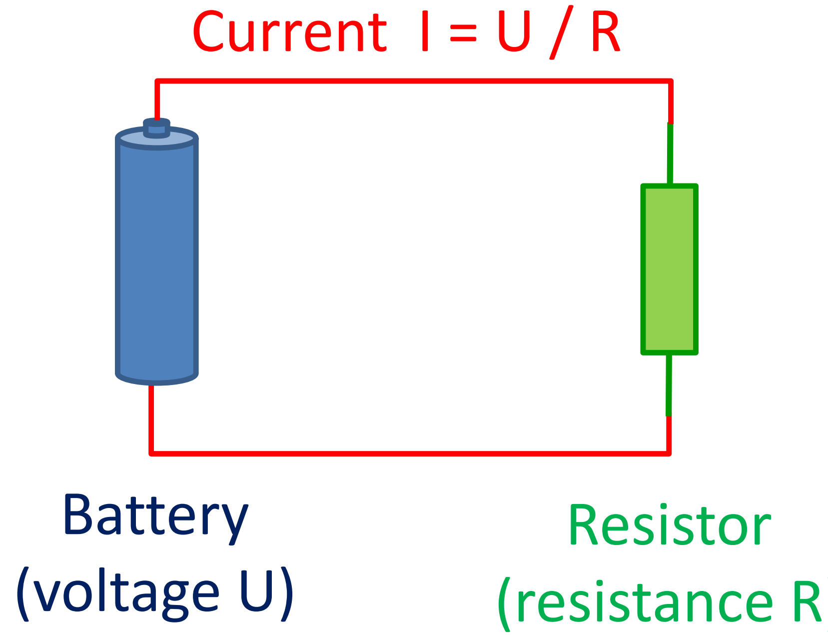
What does one need to discover the Ohm's law?



Ohm’s law

What does one need to discover the Ohm’s law?

1. many different batteries

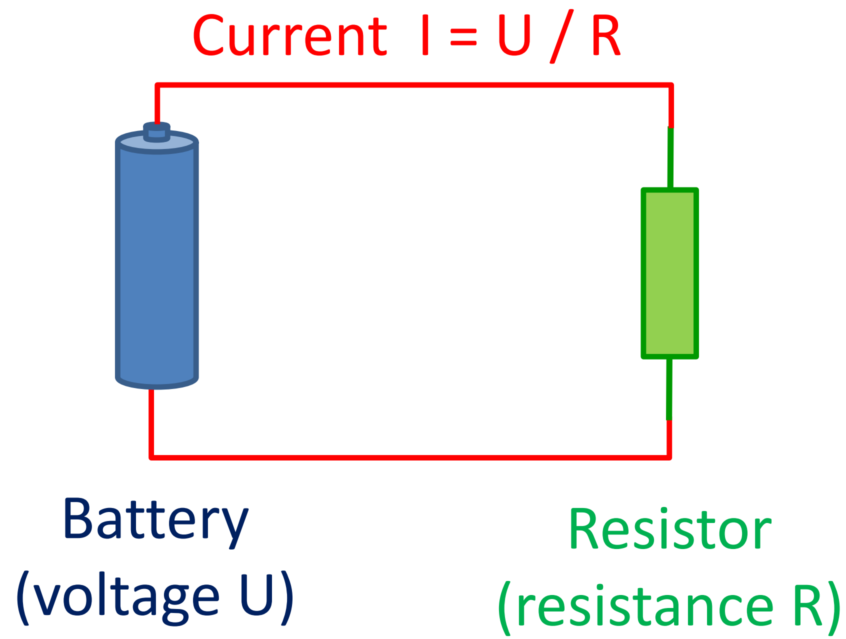


Ohm's law

What does one need to discover the Ohm's law?

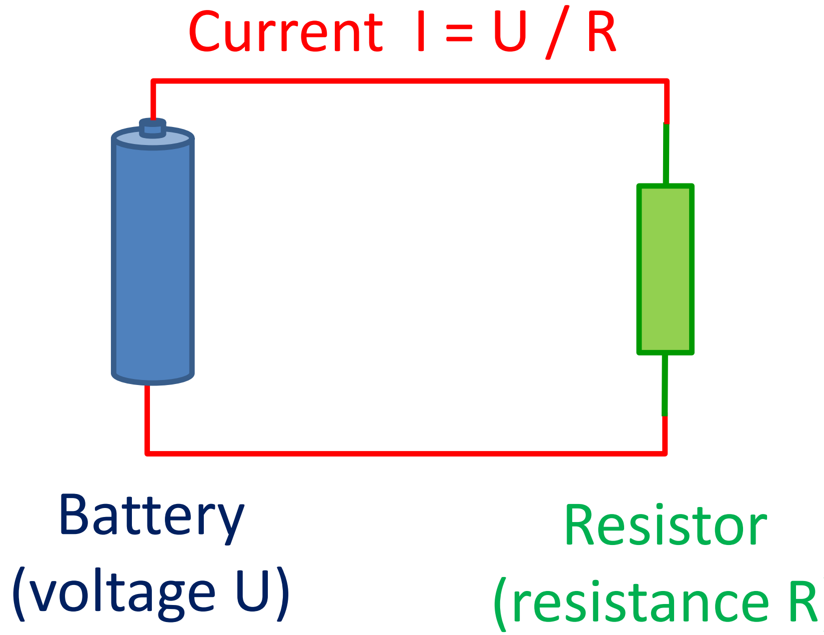
1. many different batteries

2. many different resistors



Ohm’s law

What does one need to discover the Ohm’s law?



1. many different batteries

2. many different resistors

3. ammeter (well-calibrated)

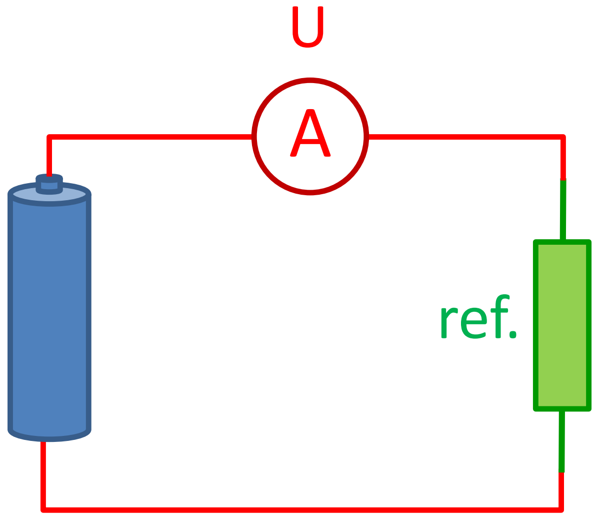
How to proceed to discover the Ohm's law?

1. select some "reference" battery and "reference" resistor

How to proceed to discover the Ohm's law?

1. select some "reference" battery and "reference" resistor

2. get the voltage U for each battery by connecting it to the "reference" resistor and measuring the current



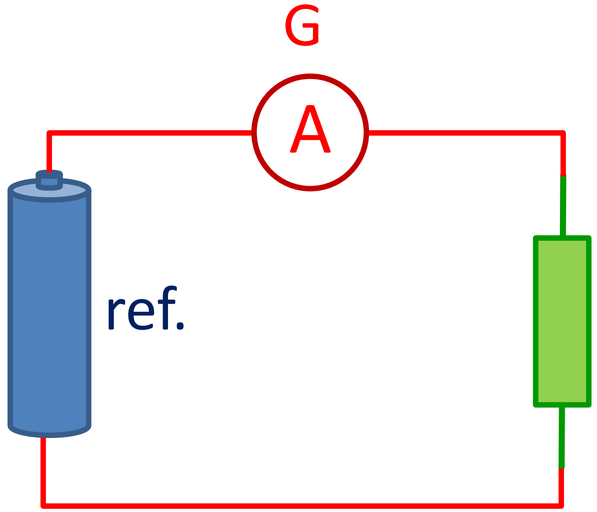
How to proceed to discover the Ohm’s law?

1. select some “reference” battery and “reference” resistor

2. get the voltage U for each battery by connecting it to the “reference” resistor and measuring the current

3. get the conductance G for each resistor by connecting it to the “reference” battery and measuring the current

I



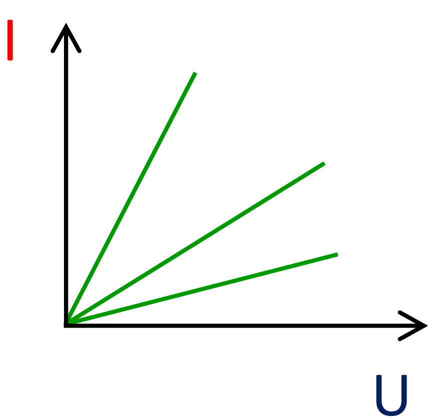
How to proceed to discover the Ohm's law?

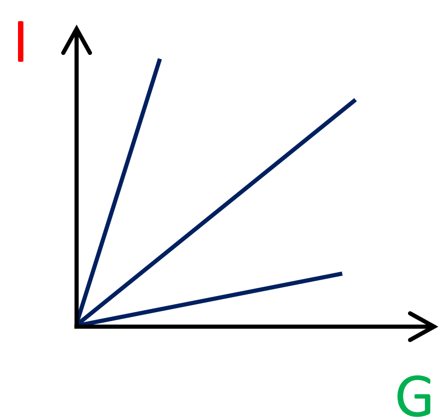
1. select some "reference" battery and "reference" resistor

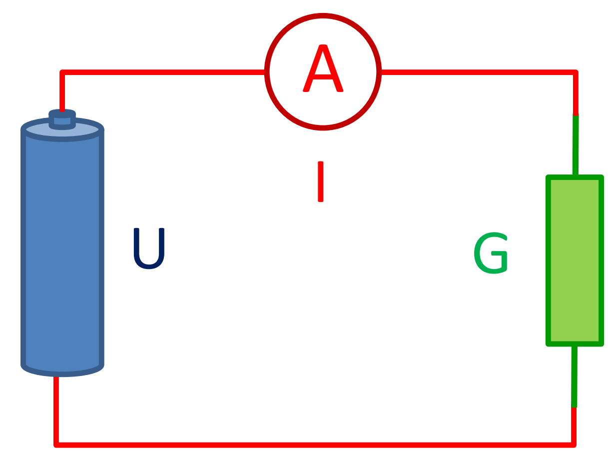
2. get the voltage U for each battery by connecting it to the "reference" resistor and measuring the current

3. get the conductance G for each resistor by connecting it to the "reference" battery and measuring the current

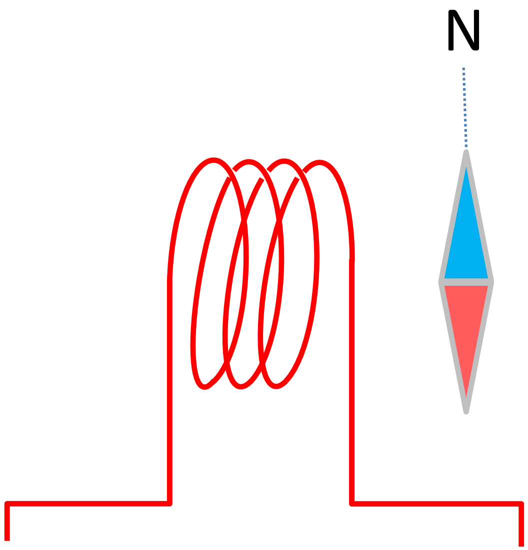
4. connect different batteries to different resistors, and discover that the current I is proportional to U and to G $\Rightarrow$ $I = U\,G$.



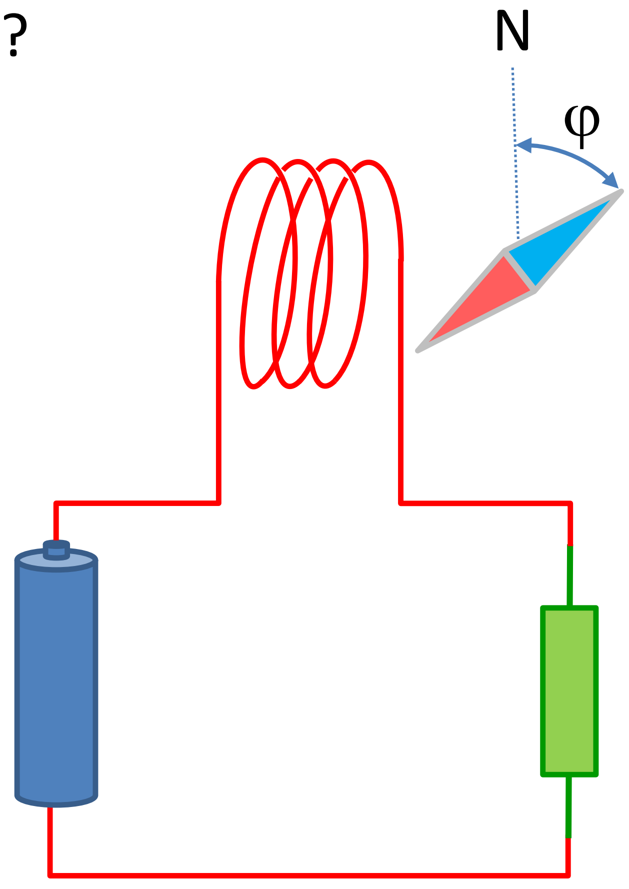




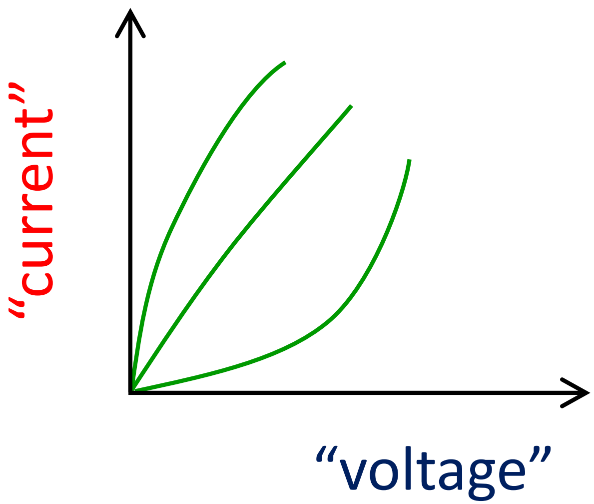
What if the ammeter is not calibrated
(and we don't know how to calibrate it)?

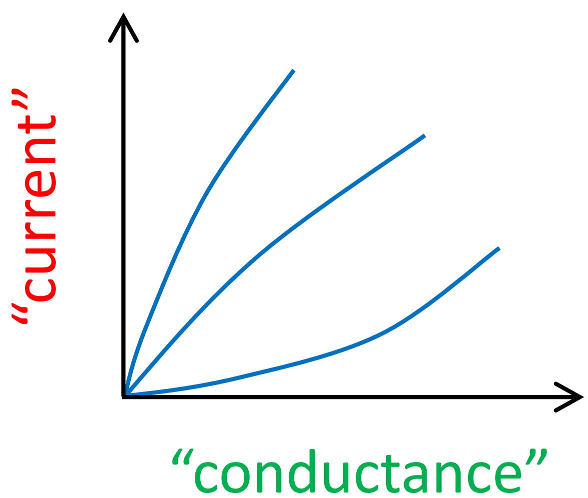


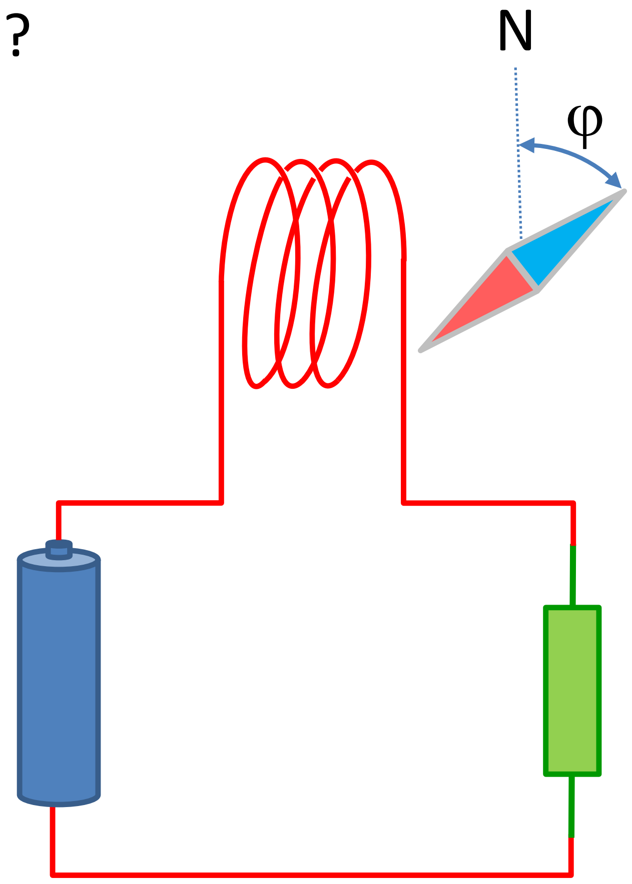
What if the ammeter is not calibrated (and we don't know how to calibrate it)?



What if the ammeter is not calibrated (and we don't know how to calibrate it)?







Nonlinear dependencies $\Rightarrow$ not clear whether there is a **linear** law behind them

Resistors

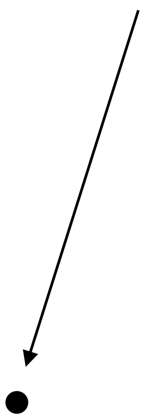
Batteries

	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

Resistors

Batteries	1	2	3	4	5	$\cdots$
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
$\vdots$						

$$\left(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}\right)$$

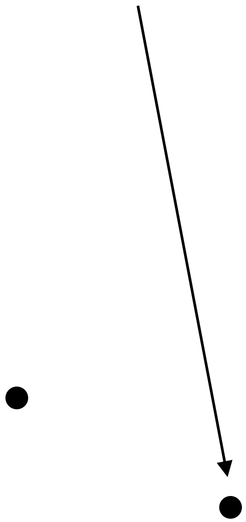


Resistors

Batteries

	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

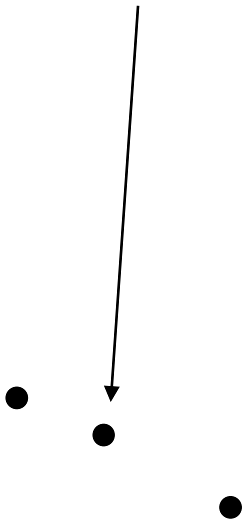
$$\left(\varphi_{11}, \varphi_{13}, \varphi_{21}, \varphi_{23}\right)$$



Resistors

Batteries	1	2	3	4	5	$\cdots$
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
$\vdots$						

$$\left(\varphi_{11}, \varphi_{14}, \varphi_{21}, \varphi_{24}\right)$$

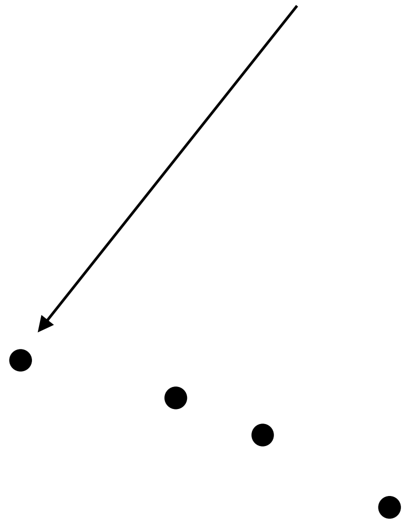


Resistors

Batteries

	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

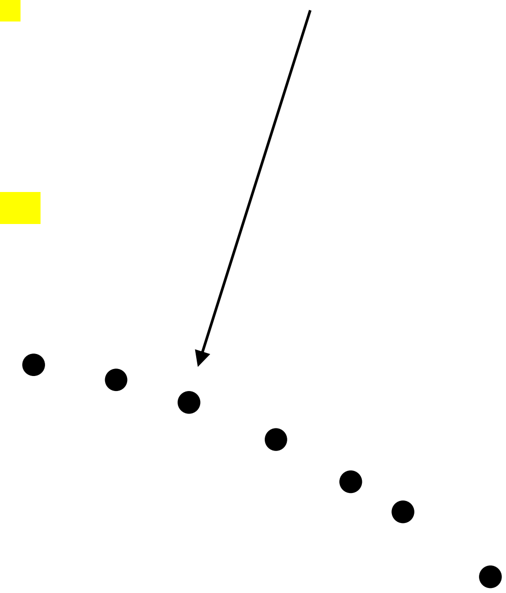
$$\left(\varphi_{11}, \varphi_{12}, \varphi_{31}, \varphi_{32}\right)$$



Resistors

Batteries	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

$$\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{j\alpha}, \varphi_{j\beta}\right)$$

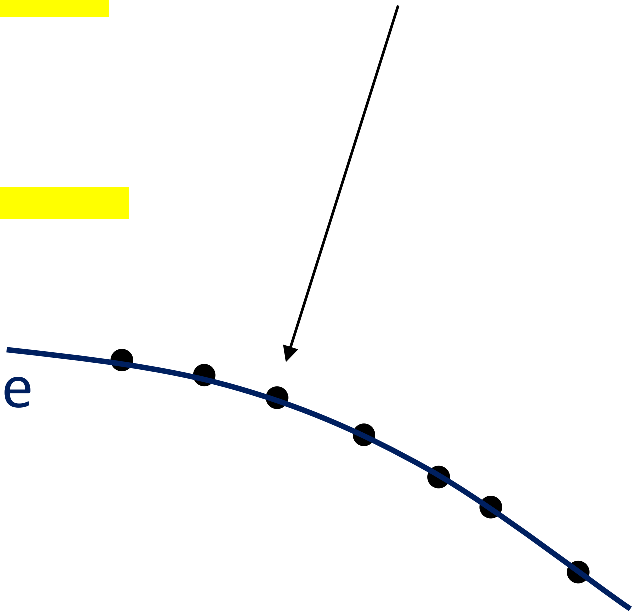


Resistors

Batteries

	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

$$\left(\varphi_{i\alpha},\varphi_{i\beta},\varphi_{j\alpha},\varphi_{j\beta}\right)$$



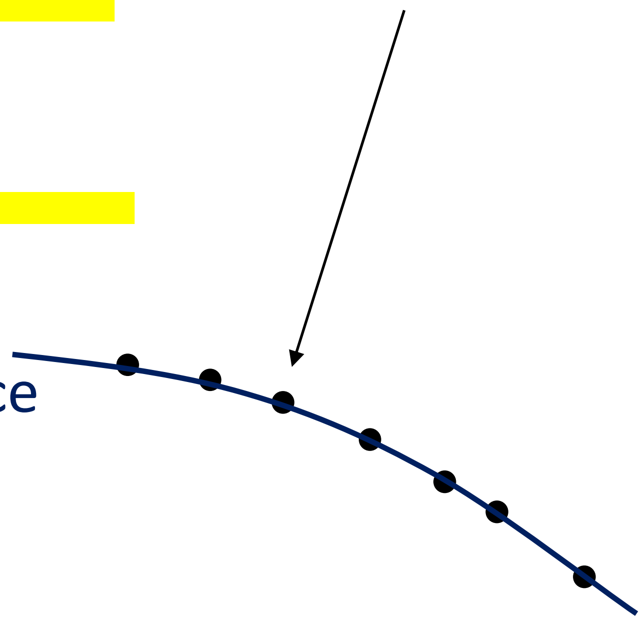
3D hyper-surface in 4D space

Resistors

Batteries

	1	2	3	4	5	$\cdots$
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
$\vdots$						

$$\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{j\alpha}, \varphi_{j\beta}\right)$$



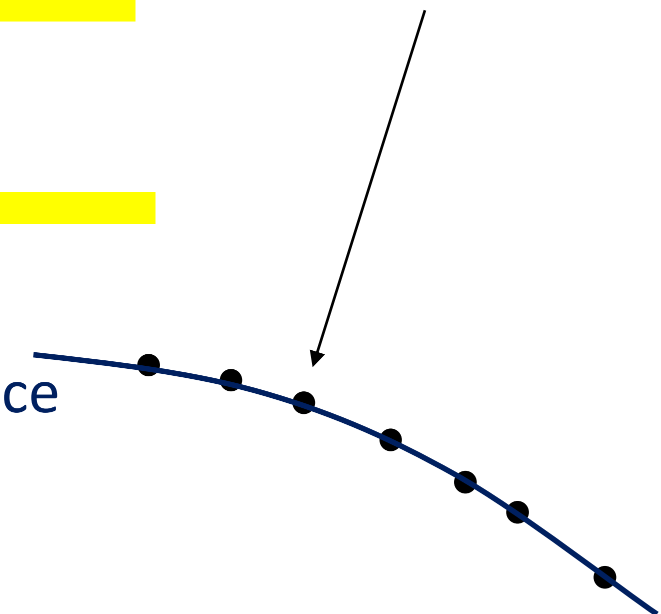
3D hyper-surface
in 4D space

$$\forall i, j, \alpha, \beta \quad \Phi\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{j\alpha}, \varphi_{j\beta}\right) = 0$$

Resistors

Batteries	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

$\left(\varphi_{i\alpha},\varphi_{i\beta},\varphi_{j\alpha},\varphi_{j\beta}\right)$



3D hyper-surface in 4D space

$$\forall i, j, \alpha, \beta \quad \Phi\left(\varphi_{i\alpha},\varphi_{i\beta},\varphi_{j\alpha},\varphi_{j\beta}\right)=0$$

Two batteries and **two** resistors → a functional dependence between 2×2 measured quantities. This is a **physical structures of rank (2,2)**.

Resistors

Batteries

	1	2	3	4	5	…
1	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	
2	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	
3	I_{31}	I_{32}	I_{33}	I_{34}	I_{35}	
4	I_{41}	I_{42}	I_{43}	I_{44}	I_{45}	
5	I_{51}	I_{52}	I_{53}	I_{54}	I_{55}	
⋮						

Resistors

Batteries

	1	2	3	4	5	…
1	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	
2	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	
3	I_{31}	I_{32}	I_{33}	I_{34}	I_{35}	
4	I_{41}	I_{42}	I_{43}	I_{44}	I_{45}	
5	I_{51}	I_{52}	I_{53}	I_{54}	I_{55}	
⋮						

$$\begin{vmatrix} I_{i\alpha} & I_{i\beta} \\ I_{j\alpha} & I_{j\beta} \end{vmatrix} = \begin{vmatrix} U_i G_\alpha & U_i G_\beta \\ U_j G_\alpha & U_j G_\beta \end{vmatrix} = 0$$

Resistors

Batteries

	1	2	3	4	5	…
1	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	
2	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	
3	I_{31}	I_{32}	I_{33}	I_{34}	I_{35}	
4	I_{41}	I_{42}	I_{43}	I_{44}	I_{45}	
5	I_{51}	I_{52}	I_{53}	I_{54}	I_{55}	
⋮						

$$\begin{vmatrix} I_{i\alpha} & I_{i\beta} \\ I_{j\alpha} & I_{j\beta} \end{vmatrix} = \begin{vmatrix} U_i G_\alpha & U_i G_\beta \\ U_j G_\alpha & U_j G_\beta \end{vmatrix} = 0$$

$$\varphi_{i\alpha} = f\left(U_i G_\alpha\right)$$

$$\Phi\left(a,b,c,d\right) \equiv \begin{vmatrix} f^{-1}(a) & f^{-1}(b) \\ f^{-1}(c) & f^{-1}(d) \end{vmatrix}$$

Resistors

Batteries	1	2	3	4	5	…
1	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	
2	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	
3	I_{31}	I_{32}	I_{33}	I_{34}	I_{35}	
4	I_{41}	I_{42}	I_{43}	I_{44}	I_{45}	
5	I_{51}	I_{52}	I_{53}	I_{54}	I_{55}	
⋮						

$$\begin{vmatrix} I_{i\alpha} & I_{i\beta} \\ I_{j\alpha} & I_{j\beta} \end{vmatrix} = \begin{vmatrix} U_i G_\alpha & U_i G_\beta \\ U_j G_\alpha & U_j G_\beta \end{vmatrix} = 0$$

$$\Phi\left(a,b,c,d\right) \equiv \begin{vmatrix} f^{-1}(a) & f^{-1}(b) \\ f^{-1}(c) & f^{-1}(d) \end{vmatrix}$$

$$\varphi_{i\alpha} = f\left(U_i G_\alpha\right)$$

$$\Downarrow$$

$$\forall i, j, \alpha, \beta \quad \Phi\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{j\alpha}, \varphi_{j\beta}\right) = 0$$

Resistors

Batteries	1	2	3	4	5	…
1	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	
2	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	
3	I_{31}	I_{32}	I_{33}	I_{34}	I_{35}	
4	I_{41}	I_{42}	I_{43}	I_{44}	I_{45}	
5	I_{51}	I_{52}	I_{53}	I_{54}	I_{55}	
⋮						

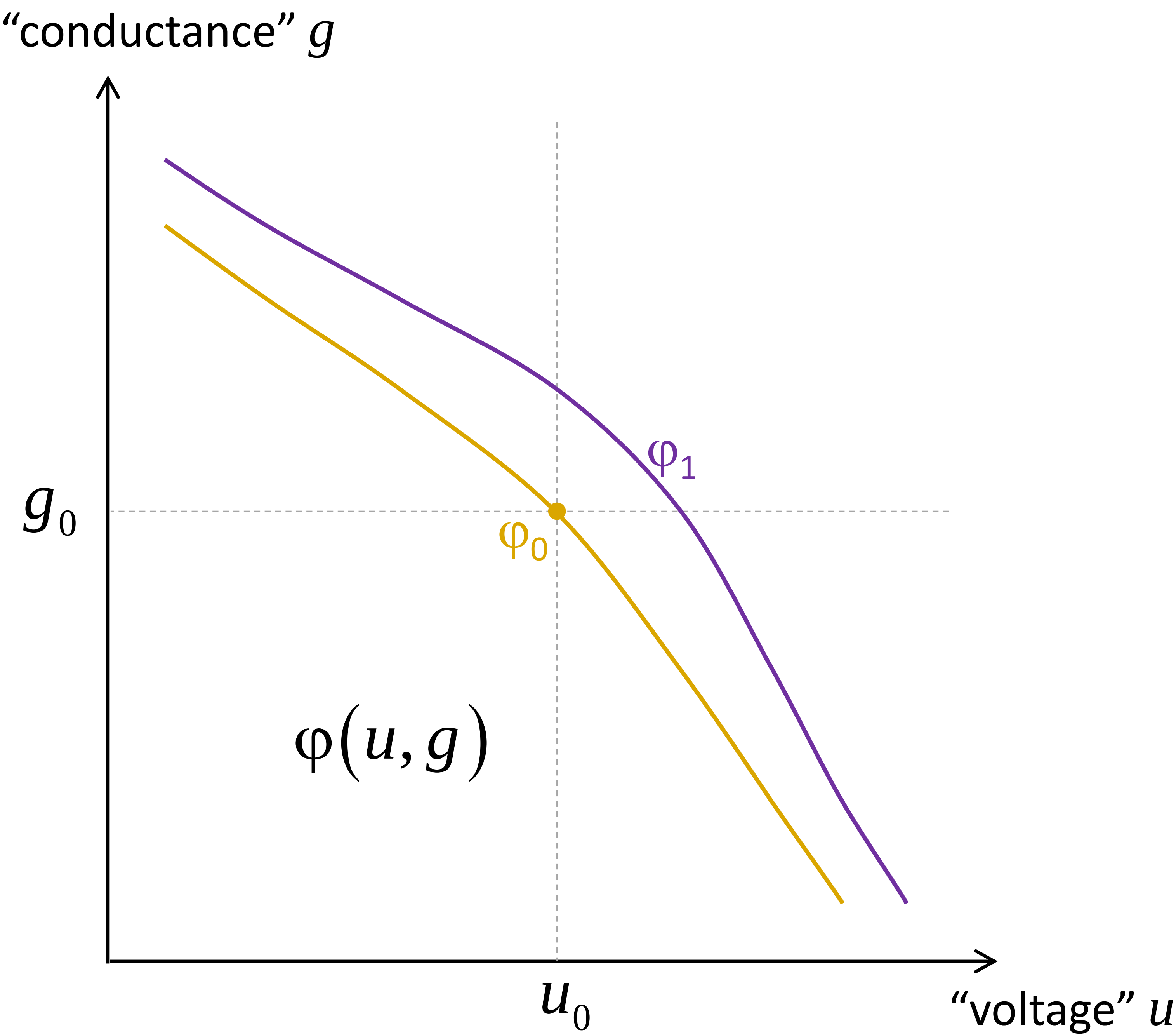
$$\begin{vmatrix} I_{i\alpha} & I_{i\beta} \\ I_{j\alpha} & I_{j\beta} \end{vmatrix} = \begin{vmatrix} U_i G_\alpha & U_i G_\beta \\ U_j G_\alpha & U_j G_\beta \end{vmatrix} = 0$$

$$\Phi\left(a,b,c,d\right) \equiv \begin{vmatrix} f^{-1}(a) & f^{-1}(b) \\ f^{-1}(c) & f^{-1}(d) \end{vmatrix}$$

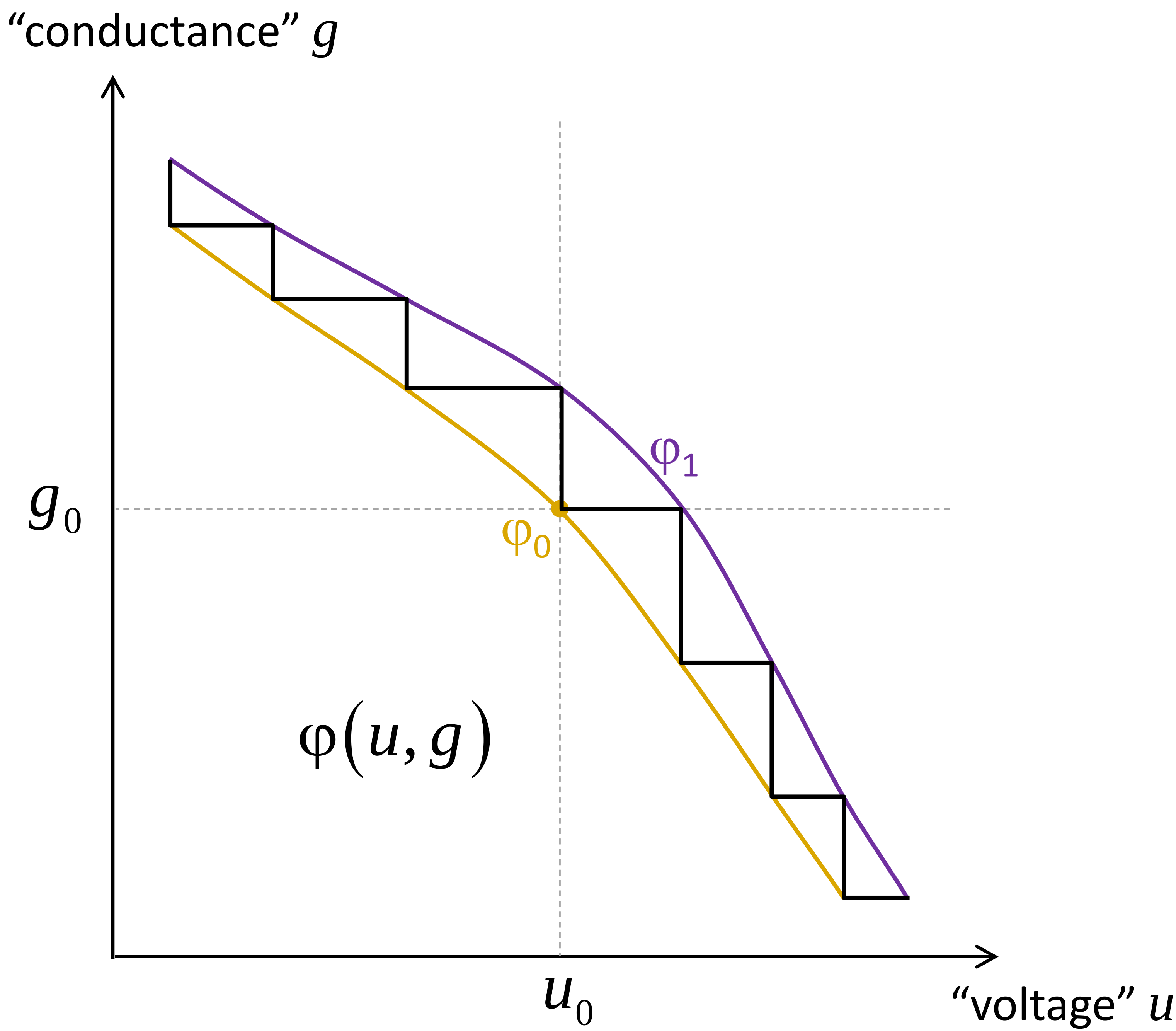
$$\varphi_{i\alpha} = f\left(U_i G_\alpha\right)$$

$$\Downarrow \quad \Uparrow$$

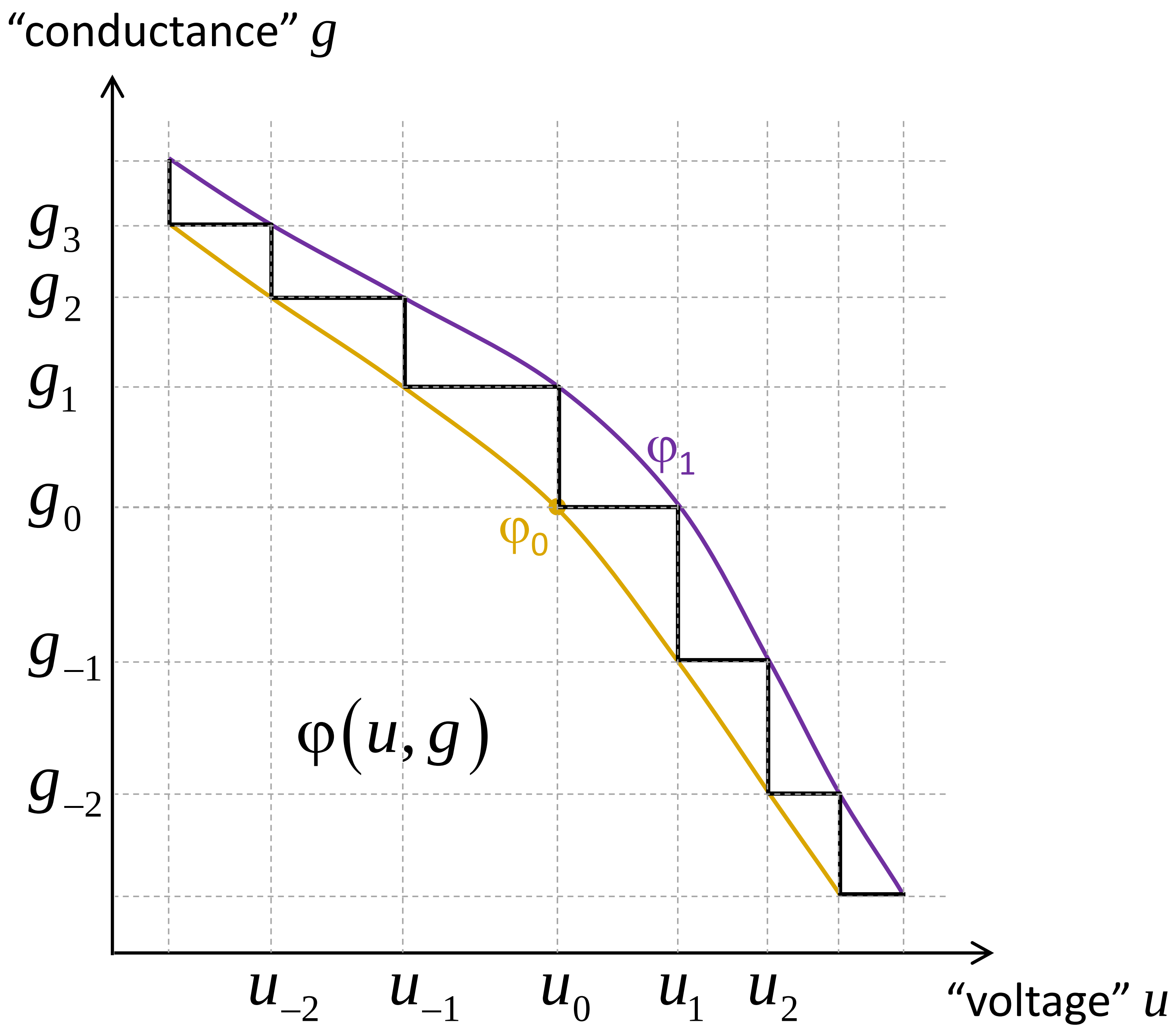
$$\forall i, j, \alpha, \beta \quad \Phi\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{j\alpha}, \varphi_{j\beta}\right) = 0$$



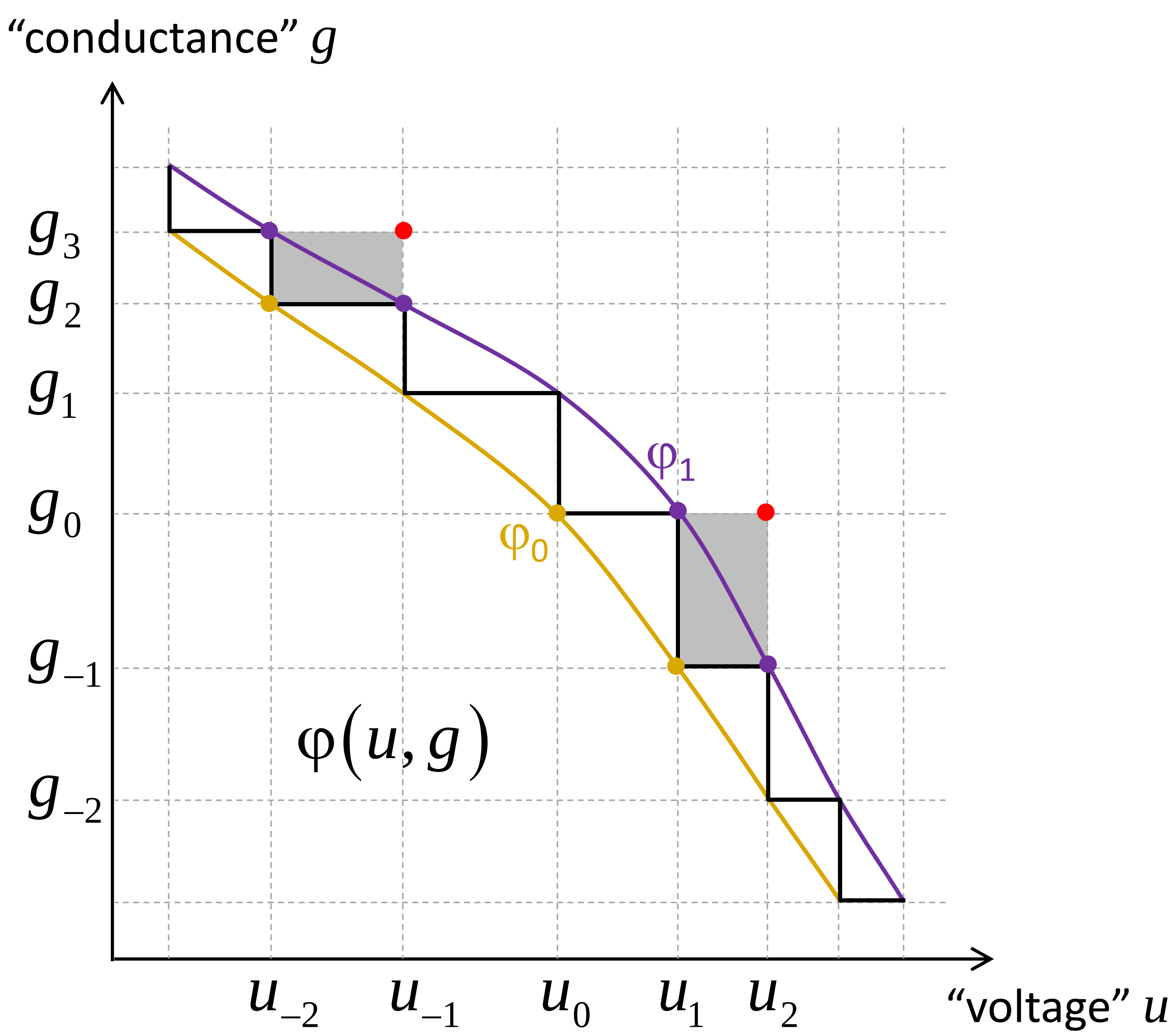
“conductance” g
g_0
φ_1
φ_0
$\varphi(u, g)$
u_0
“voltage” u



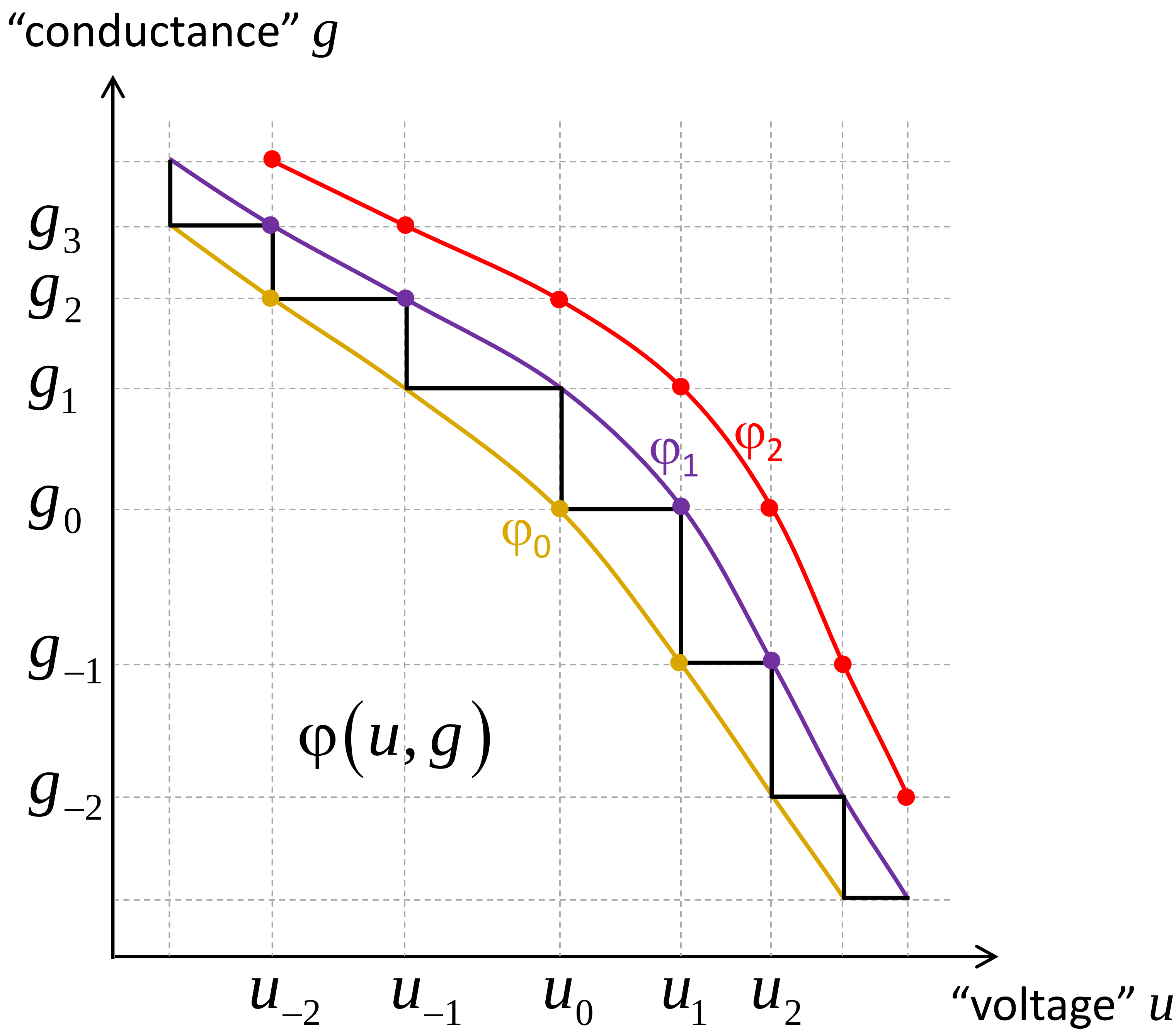
“conductance” g
φ1
g0
φ0
φ(u, g)
u0
“voltage” u



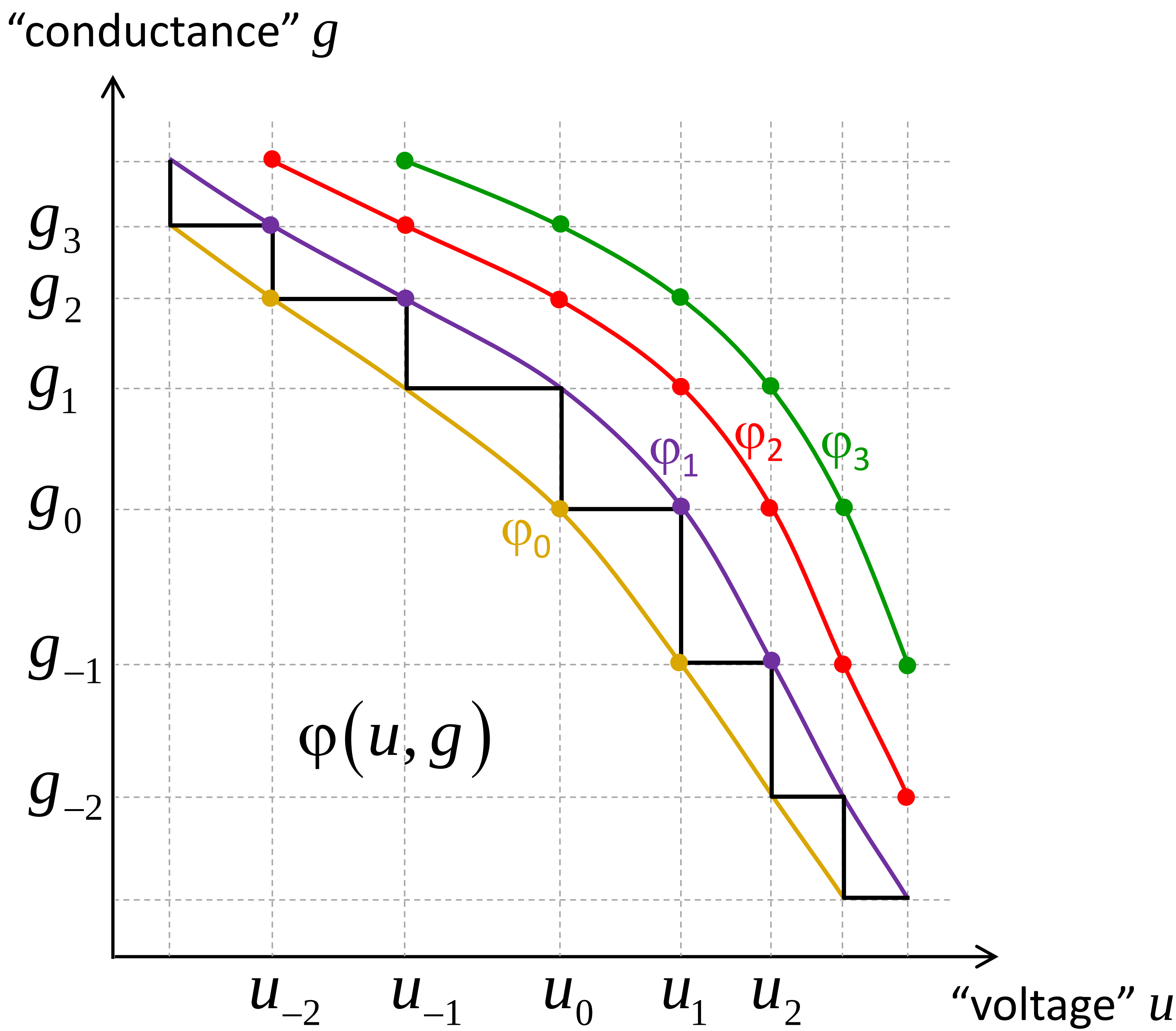
“conductance” g
g_3
g_2
g_1
g_0
g_−1
g_−2
φ_1
φ_0
φ(u, g)
u_−2
u_−1
u_0
u_1
u_2
“voltage” u



“conductance” g
g_3
g_2
g_1
g_0
g_{-1}
g_{-2}
φ_1
φ_0
$\varphi(u, g)$
u_{-2}
u_{-1}
u_0
u_1
u_2
“voltage” u

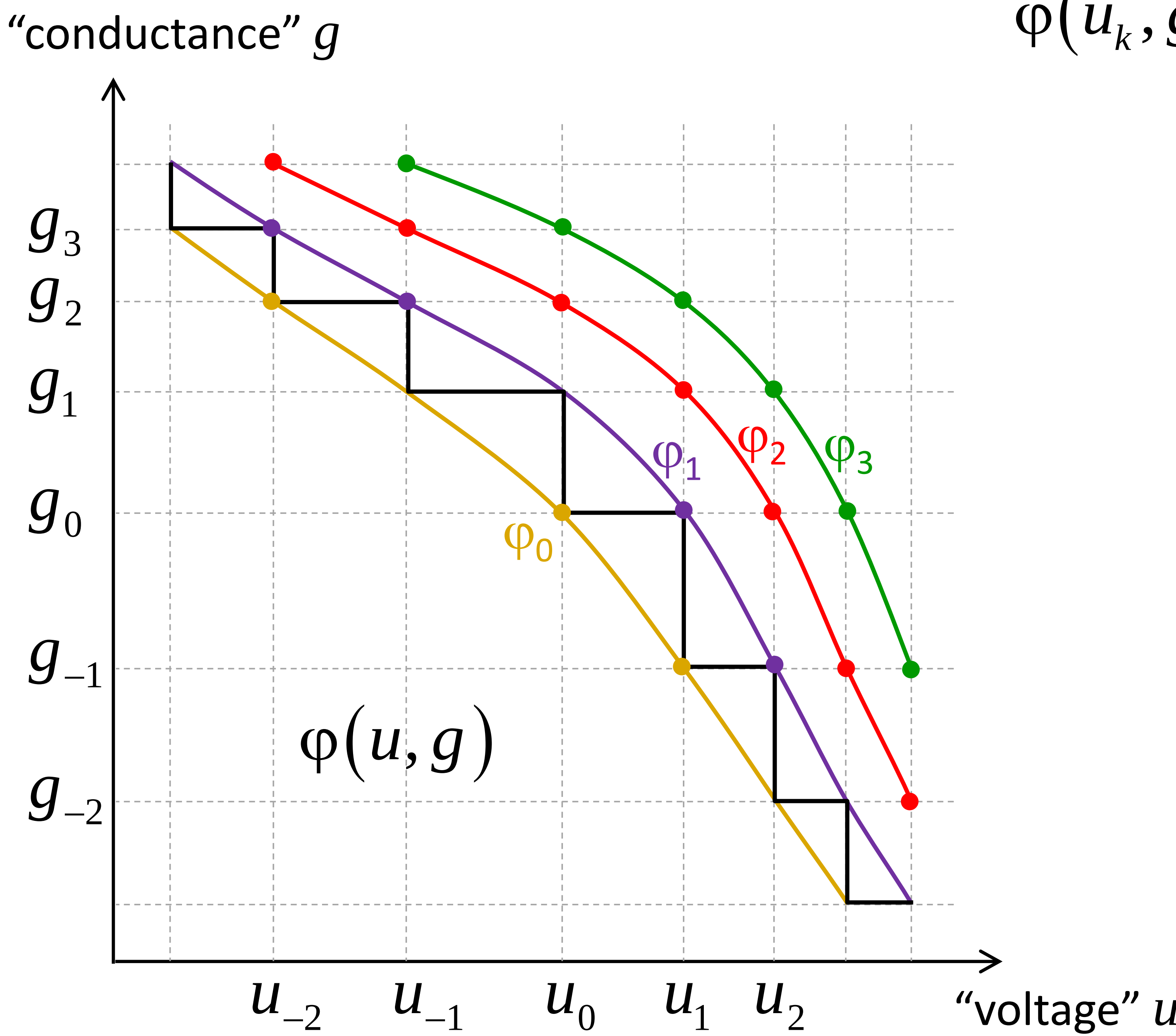


“conductance” g
g_3
g_2
g_1
g_0
g_{-1}
g_{-2}
φ_1
φ_2
φ_0
$\varphi(u, g)$
u_{-2}
u_{-1}
u_0
u_1
u_2
“voltage” u

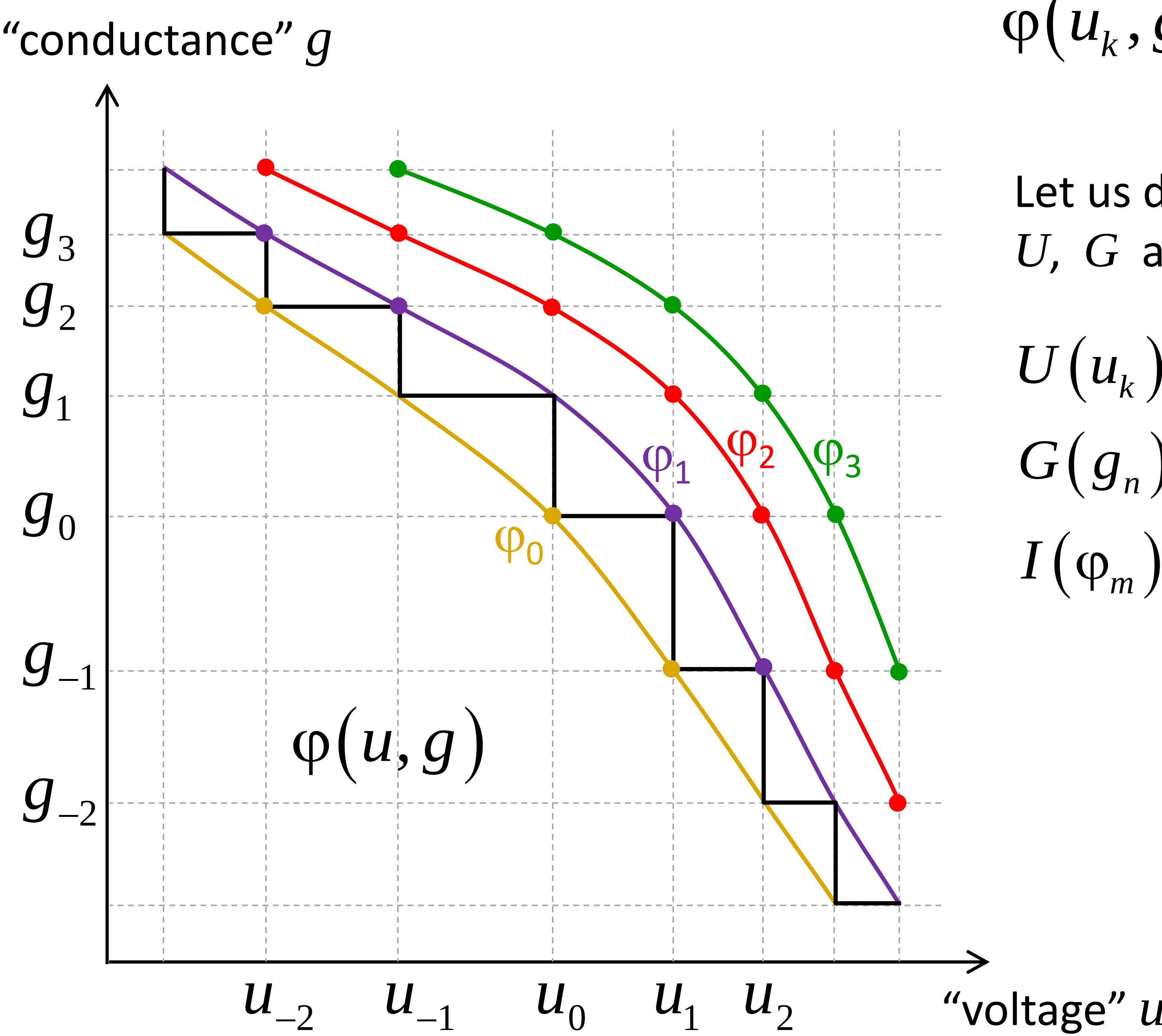


“conductance” g
g_3
g_2
g_1
g_0
g_−1
g_−2
φ_1
φ_2
φ_3
φ_0
φ(u, g)
u_−2
u_−1
u_0
u_1
u_2
“voltage” u

$$\varphi(u_k, g_n) = \varphi_{k+n}$$



$$\varphi\left(u_k, g_n\right) = \varphi_{k+n}$$

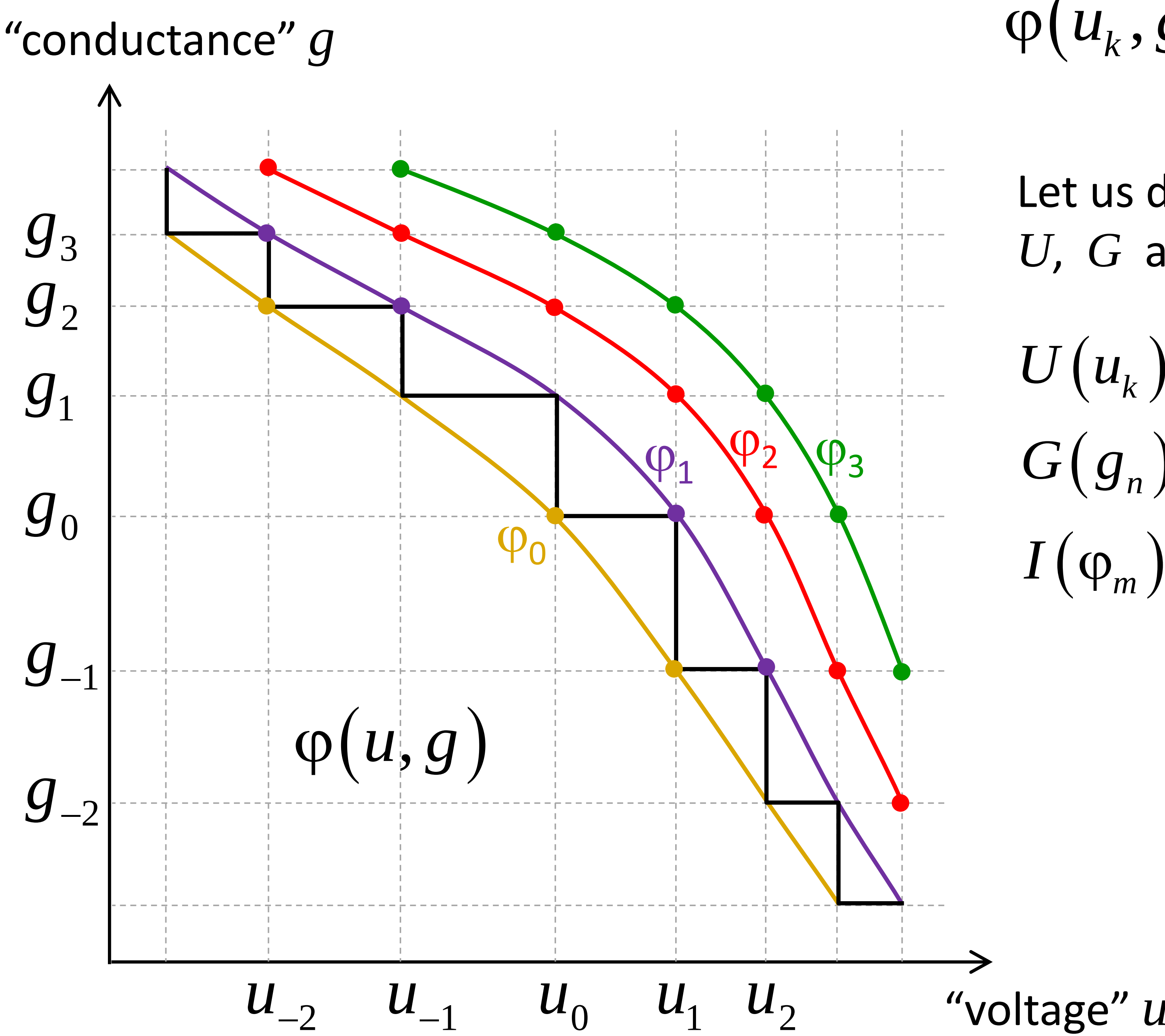


Let us define U, G and I as

$$U\left(u_k\right) = e^k$$

$$G\left(g_n\right) = e^n$$

$$I\left(\varphi_m\right) = e^m$$



$$\varphi(u_k, g_n) = \varphi_{k+n}$$

Let us define U, G and I as

$$U(u_k) = e^k$$

$$G(g_n) = e^n$$

$$I(\varphi_m) = e^m$$

$$I = U\,G$$

F = ma: law or definition?

$$F_i = m_\alpha a_{i\alpha}$$

usual formulation of Newton’s second law

F = ma: law or definition?

$$F_i = m_\alpha a_{i\alpha}$$

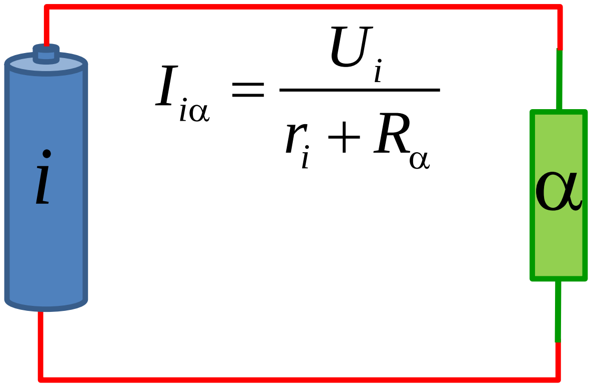
usual formulation of Newton's second law

$$\Updownarrow$$

$$\begin{vmatrix} a_{i\alpha} & a_{i\beta} \\ a_{j\alpha} & a_{j\beta} \end{vmatrix} = 0$$

Newton's second law as a physical structure of rank (2,2)

Once again about Ohm's law: physical structure of rank (2,3)



$$I_{i\alpha} = \frac{U_i}{r_i + R_\alpha}$$

Battery
(e.m.f. U_i and inner resistance r_i)

Resistor
(resistance R_α)

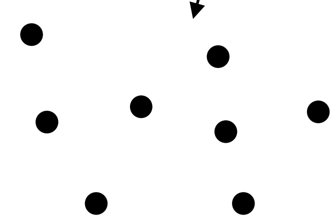
Once again about Ohm's law: physical structure of rank (2,3)

Resistors

Batteries	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

$$\varphi_{i\alpha} = f\left(I_{i\alpha}\right) \equiv f\left(\frac{U_i}{r_i + R_\alpha}\right)$$

$$\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{j\alpha}, \varphi_{j\beta}\right)$$



points are everywhere in 4D space $\Rightarrow$ no physical structure of rank (2,2)

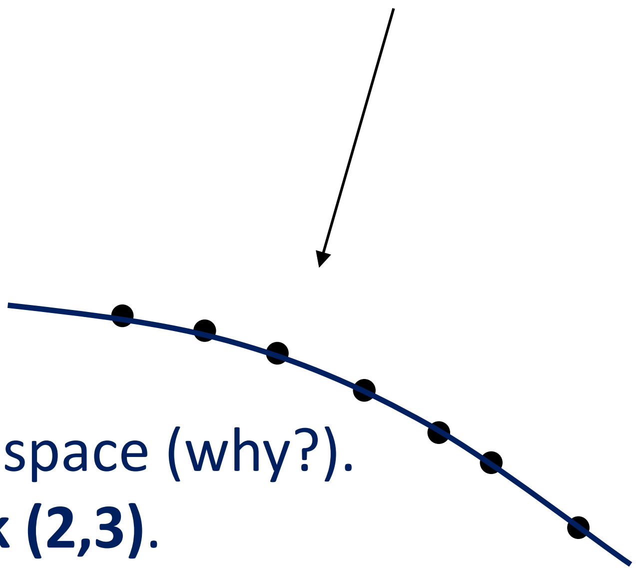
Once again about Ohm's law: physical structure of rank (2,3)

Resistors (columns) / Batteries (rows)

	1	2	3	4	5	…
1	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	
2	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	
3	φ_{31}	φ_{32}	φ_{33}	φ_{34}	φ_{35}	
4	φ_{41}	φ_{42}	φ_{43}	φ_{44}	φ_{45}	
5	φ_{51}	φ_{52}	φ_{53}	φ_{54}	φ_{55}	
⋮						

$$\varphi_{i\alpha} = f\left(I_{i\alpha}\right) \equiv f\left(\frac{U_i}{r_i + R_\alpha}\right)$$

$$\left(\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{i\gamma}, \varphi_{j\alpha}, \varphi_{j\beta}, \varphi_{j\gamma}\right)$$



All points are in a 5D hyper-surface of 6D space (why?).

This is a **physical structure of rank (2,3)**.

Once again about Ohm's law: physical structure of rank (2,3)

$$I_{i\alpha} = \frac{U_i}{r_i + R_\alpha}$$

$$M_1 = \begin{pmatrix} 0 & U_i^{-1} & r_i/U_i \\ 0 & U_j^{-1} & r_j/U_j \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 1 & 1 \\ R_\alpha & R_\beta & R_\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

Once again about Ohm's law: physical structure of rank (2,3)

$$I_{i\alpha} = \frac{U_i}{r_i + R_\alpha}$$

$$M_1 = \begin{pmatrix} 0 & U_i^{-1} & r_i / U_i \\ 0 & U_j^{-1} & r_j / U_j \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 1 & 1 \\ R_\alpha & R_\beta & R_\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

$$M_1 M_2 = \begin{pmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{pmatrix}$$

Once again about Ohm's law: physical structure of rank (2,3)

$$I_{i\alpha} = \frac{U_i}{r_i + R_\alpha} \qquad M_1 = \begin{pmatrix} 0 & U_i^{-1} & r_i/U_i \\ 0 & U_j^{-1} & r_j/U_j \\ 0 & 0 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & 1 & 1 \\ R_\alpha & R_\beta & R_\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

$$M_1 M_2 = \begin{pmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{pmatrix}$$

$$\det\left(M_1\right) = \det\left(M_2\right) = 0 \quad \Rightarrow \quad \begin{vmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Once again about Ohm's law: physical structure of rank (2,3)

$$I_{i\alpha} = \frac{U_i}{r_i + R_\alpha} \qquad M_1 = \begin{pmatrix} 0 & U_i^{-1} & r_i/U_i \\ 0 & U_j^{-1} & r_j/U_j \\ 0 & 0 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & 1 & 1 \\ R_\alpha & R_\beta & R_\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

$$M_1 M_2 = \begin{pmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{pmatrix}$$

$$\det(M_1) = \det(M_2) = 0 \quad \Rightarrow \quad \begin{vmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\varphi_{i\alpha} = f\left(\frac{U_i}{r_i + R_\alpha}\right)$$

⇒ six quantities $\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{i\gamma}, \varphi_{j\alpha}, \varphi_{j\beta}, \varphi_{j\gamma}$ are not independent of each other

Once again about Ohm's law: physical structure of rank (2,3)

$$I_{i\alpha} = \frac{U_i}{r_i + R_\alpha} \qquad M_1 = \begin{pmatrix} 0 & U_i^{-1} & r_i/U_i \\ 0 & U_j^{-1} & r_j/U_j \\ 0 & 0 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & 1 & 1 \\ R_\alpha & R_\beta & R_\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

$$M_1 M_2 = \begin{pmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{pmatrix}$$

$$\det(M_1) = \det(M_2) = 0 \quad \Rightarrow \quad \begin{vmatrix} I_{i\alpha}^{-1} & I_{i\beta}^{-1} & I_{i\gamma}^{-1} \\ I_{j\alpha}^{-1} & I_{j\beta}^{-1} & I_{j\gamma}^{-1} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\varphi_{i\alpha} = f\left(\frac{U_i}{r_i + R_\alpha}\right) \iff$$ six quantities $\varphi_{i\alpha}, \varphi_{i\beta}, \varphi_{i\gamma}, \varphi_{j\alpha}, \varphi_{j\beta}, \varphi_{j\gamma}$ are not independent of each other

Examples of functional equations

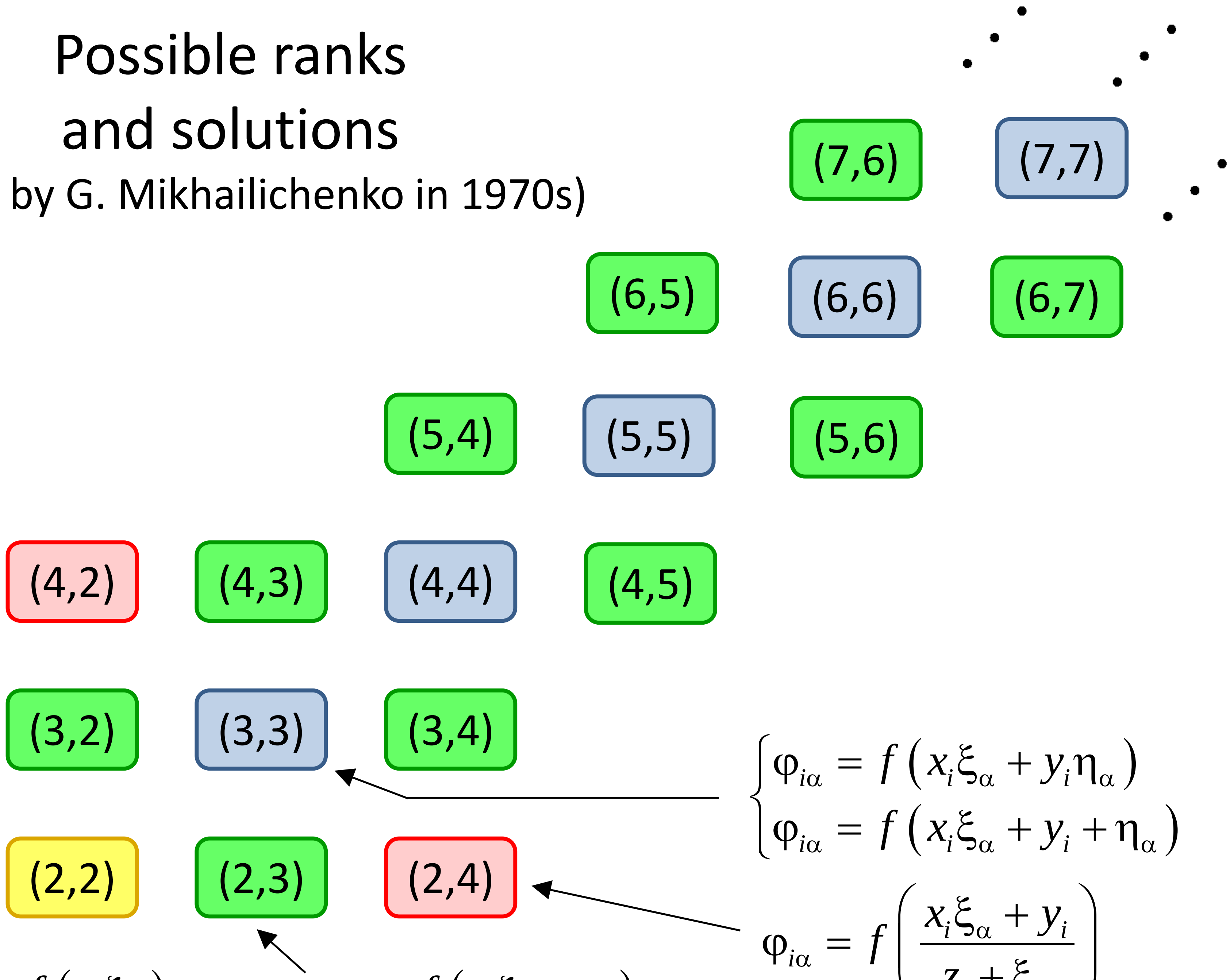
equation	**solution**
$f(x+y) = f(x) + f(y)$	$f(x) = C \cdot x$
$f(x+y) = f(x) \cdot f(y)$	$f(x) = \exp(C \cdot x)$
$\begin{cases} f(x-y) = f(x) \cdot f(y) + g(x) \cdot g(y) \\ 0 < x\, f(x) < g(x) < x \ \text{ for } \ 0 < x < 1 \end{cases}$	$f(x) = \cos x, \quad g(x) = \sin x$
$\Phi\big(\varphi(x_1, \xi_1), \ \varphi(x_1, \xi_2), \ \varphi(x_2, \xi_1), \ \varphi(x_2, \xi_2) \big) = 0$	$\varphi(x, \xi) = f(x\xi)$ or, equivalently, $\varphi(x, \xi) = f(x + \xi)$

Functional equations of the Theory of physical structures

rank	equation	solution
(2,2)	$\Phi\begin{pmatrix} \varphi_{i\alpha}, & \varphi_{i\beta}, \\ \varphi_{j\alpha}, & \varphi_{j\beta} \end{pmatrix} = 0$	$\varphi_{i\alpha} = f\left(x_i \xi_\alpha\right)$ or, equivalently, $\varphi_{i\alpha} = f\left(x_i + \xi_\alpha\right)$
(2,3)	$\Phi\begin{pmatrix} \varphi_{i\alpha}, & \varphi_{i\beta}, & \varphi_{i\gamma}, \\ \varphi_{j\alpha}, & \varphi_{j\beta}, & \varphi_{j\gamma} \end{pmatrix} = 0$	$\varphi_{i\alpha} = f\left(x_i \xi_\alpha + y_i\right)$
(2,4)	$\Phi\begin{pmatrix} \varphi_{i\alpha}, & \varphi_{i\beta}, & \varphi_{i\gamma}, & \varphi_{i\delta}, \\ \varphi_{j\alpha}, & \varphi_{j\beta}, & \varphi_{j\gamma}, & \varphi_{j\delta} \end{pmatrix} = 0$	$\varphi_{i\alpha} = f\left(\dfrac{x_i \xi_\alpha + y_i}{z_i + \xi_\alpha}\right)$
(3,3)	$\Phi\begin{pmatrix} \varphi_{i\alpha}, & \varphi_{i\beta}, & \varphi_{i\gamma}, \\ \varphi_{j\alpha}, & \varphi_{j\beta}, & \varphi_{j\gamma}, \\ \varphi_{k\alpha}, & \varphi_{k\beta}, & \varphi_{k\gamma} \end{pmatrix} = 0$	$\varphi_{i\alpha} = f\left(x_i \xi_\alpha + y_i \eta_\alpha\right)$ or $\varphi_{i\alpha} = f\left(x_i \xi_\alpha + y_i + \eta_\alpha\right)$

Possible ranks and solutions

(found by G. Mikhailichenko in 1970s)



$$\begin{cases} \varphi_{i\alpha} = f\left(x_i\xi_\alpha + y_i\eta_\alpha\right) \\ \varphi_{i\alpha} = f\left(x_i\xi_\alpha + y_i + \eta_\alpha\right) \end{cases}$$

$$\varphi_{i\alpha} = f\left(\frac{x_i\xi_\alpha + y_i}{z_i + \xi_\alpha}\right)$$

$$\varphi_{i\alpha} = f\left(x_i\xi_\alpha\right)$$

$$\varphi_{i\alpha} = f\left(x_i\xi_\alpha + y_i\right)$$

Some works on theory of physical structures

- Yu. I. Kulakov, Theory of physical structures, Journal of Soviet Mathematics, v. 27, pp. 2616-2646 (1984)   (review of results and possible applications).
- G. G. Mikhailichenko, A problem in the theory of physical structures, Siberian Mathematical Journal, v. 18, pp. 951-961 (1977)   (proofs for ranks (2,2), (2,3), (2,4)).
- G. G. Mikhailichenko, On a functional equation with two-index variables, Ukrainian Mathematical Journal, v. 25, pp. 490-497 (1973)   (proofs for ranks (3,3), (3,4)).