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Monopoly Regulation Through Cost-Based Revenue Caps Employing A Base Year Approach

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Abstract

This paper studies the pricing incentives of a monopolist constrained by a revenue cap endogenously determined by her costs in a so-called base year. Such regulation is employed, among others, to govern electricity distribution operators in Germany. We show that the revenue cap may incentivize excessive supply in the base year to reap profits in the non-base years. A connected set of price caps exists so that a hybrid regulation consisting of any element in this set and the cost-based revenue cap unambiguously improves welfare and, under some conditions, even leads to the socially optimal outcome.

Keywords: regulation, monopoly, electricity distribution

JEL Classification Codes: D21, D42, L12, L51, L94

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1 Introduction

Distribution network operators (DSOs) responsible for electricity distribution form natural monopolies (e.g., Crew and Kleindorfer, 1986; Kahn, 1988; Sherman, 1989; Joskow, 2007). While the necessity for regulation is widely acknowledged, determining the appropriate form remains a challenge for authorities (see, e.g., Law, 1995; Cowan, 1997). The German Incentive Regulation Ordinance (ARegV) employs a cost-based revenue cap, in which a regulatory period consists of one base year and four non-base years. Each year's revenue is subject to a cap endogenously determined by the DSO's costs in the last regulatory period's base year (Cullmann and Nieswand, 2016; Matschoss et al., 2019).[1] The main goal of the ARegV is to introduce incentives for DSOs to engage in cost-reducing and, hence, welfare-enhancing actions (Matschoss et al., 2019). Although the regulation is likely efficacious in that way, as revenue caps generally are (Laffont and Tirole, 1993; Crew and Kleindorfer, 1996), it is not as straightforward how it performs in other aspects. In their empirical analysis, Cullmann and Nieswand (2016) show that the regulation led to incentives for inflating costs in the base years by overinvesting, shifting investments, and excessive spending. By now, the ARegV thwarts these strategic investment behaviors through a combination of cost-of-service and yardstick regulation.[2] Nevertheless, DSOs can still influence the costs in the base year, and thus their revenue cap, by manipulating the quantity demanded via pricing.

This paper studies the theoretical incentives for DSOs to apply such cost manipulations and their welfare implications. To do so, we investigate the pricing strategy of a monopolist constrained by a cost-based revenue cap. The monopolist operates in one base year and T non-base years, where each year's revenue must not exceed the base year's operating costs. Under some conditions likely to hold in energy markets, a monopolist has incentives to decrease prices in the base year, enabling profit extraction in the non-base years. We show that combining the cost-based revenue cap with a price cap countervails potential adverse effects, ultimately leading to a welfare increase. Such a hybrid regulation may even lead to the socially optimal outcome. If this is the case, then there is not a unique but a connected set of price caps that do so. This property is essential for policymakers since it reduces the authorities' required information processing capacity, generally forming an issue in monopoly regulation (Laffont and Tirole, 1993).

Our findings align with the existing literature, which generally agrees that revenue cap regulation often induces further inefficiencies. Crew and Kleindorfer (1986, 1996) show that imposing a fixed revenue cap upon a monopolist facing a linear demand curve yields greater inefficiencies than the unregulated scenario. They contend that the monopolist has two approaches to address the burden of the revenue restriction: (i) she can either decrease the price and, thereby, increase overall output or (ii) increase the price and,

[1] Similar regulatory approaches, to which the results of this paper extend, are employed to govern electricity and/or gas DSOs in Austria, Croatia, Cyprus, France, Ireland, Lithuania, Luxembourg, North Macedonia, Romania, Slovenia, Sweden, and Ukraine, among others (CEER, 2023).

[2] The ARegV introduces "capital cost adjustments" in § 10a ARegV to mitigate strategic investment behavior. At the same time, DSOs are subject to yardstick regulation according to § 12 ARegV, which establishes a particular network operator's cost structure efficiency relative to other network operators (Federal Ministry for Economic Affairs and Climate Action, 2021). Since the revenue cap only accounts for efficient costs, the DSOs cannot engage in strategic cost inflation through excessive spending.

thereby, decrease overall output. Since costs increase in quantity supplied, the monopolist maximizes profits by employing the second strategy. Consequently, the revenue cap incentivizes the monopolist to charge even larger prices than under no regulation, ultimately leading to a welfare loss.

Comnes et al. (1995) generalize these results to demand functions beyond the linear case. They establish necessary conditions for the respective incentive problem to arise and induce an efficiency loss, namely: (i) the price elasticity of high energy prices is sufficiently large, (ii) total costs are an increasing function of quantity, and (iii) a revenue curve that is not only upward sloping in the short run due to the inelastic demand for electricity. Furthermore, they propose a hybrid approach of revenue and price cap regulation to solve the incentive problem: The introduction of a price cap to an existing revenue cap regulation prohibits price increases as an achievable strategy for the monopolist, leaving a price decrease as the only viable option to meet the revenue cap. At the monopolist's optimal prices under this hybrid regulation, the price cap is globally binding in that it allows small but not large price changes, while the revenue cap is binding at the margin.

Our results indicate that the necessary conditions of Comnes et al. (1995) no longer hold under revenue caps derived from the costs of a specific base year. Hence, when fixed revenue caps lead to desirable outcomes, endogenously determined revenue caps utilizing a base year approach (as employed in the ARegV) may fail to do so. Moreover, in line with Comnes et al. (1995), we propose a hybrid regulation combining revenue cap regulation with a price cap as a solution to address inefficient pricing behavior by the monopolist. The underlying intuition aligns: The introduction of a complementary price cap restricts the set of attainable strategies for the monopolist and, thereby, may render the monopolist's best available choice the socially optimal price. Consequently, joint consideration of our results and Comnes et al. (1995) suggests that the addition of a price cap can remedy several inefficiencies that may arise under revenue cap regulation, strengthening the argument for hybrid approaches over pure revenue caps.

By studying an endogenously determined revenue cap rather than a fixed one, we closely relate to the analysis by Lantz (2008). Lantz (2008) studies a monopolist who offers a non-discriminatory two-part pricing tariff and faces a two-part revenue cap consisting of a fixed and variable part, where the variable part depends on the quantity supplied. The monopolist sets the variable price component of the tariff to equal her marginal costs and the fixed part to extract the consumer surplus needed to reach the revenue cap. Similar to Crew and Kleindorfer (1986, 1996), Lantz (2008) argues that the monopolist may provide an inefficiently low quantity. For the monopolist to choose the efficient output level, Lantz (2008) finds that revenue must be allowed to increase faster than the total costs of an increase in quantity. Consequently, Lantz (2008) concludes that revenue caps are not a good idea because efficient regulation requires vast informational requirements regarding the monopolist's costs.

Similar to Crew and Kleindorfer (1986, 1996), the inefficiencies identified by Lantz (2008) stem from the functional form energy demand and the profitability of minimizing costs at a given revenue level. In contrast, our analysis focuses on the role of the base year approach in shaping the monopolist's inter-temporal optimization problem and resulting inefficiencies. In this regard, we complement the findings by Lantz (2008). Moreover, in stark contrast to Lantz (2008), we find that relatively few informational requirements are necessary to adjust the revenue cap to be (part of) an efficiency-inducing regulatory approach.

By assuming energy demand to exhibit constant elasticity, we closely align with Campbell (2018a), who provides another analysis of an endogenously determined revenue cap. In particular, each year's revenue cap is a function of the preceding year's revenue. Campbell (2018a) studies a monopolist operating in multiple markets and engaging in price discrimination across these markets. He uses a linear optimization approach to contrast the welfare implications of price and revenue cap regulation. He finds that the relative advantageousness of revenue caps increases if demand is inelastic because, as prices increase, total demand decreases less than the total cost of production. By calibrating the theoretical framework with data from the Jamaican energy market, Campbell (2018a) finds evidence of a welfare-decreasing cross-subsidization by residential consumers towards commercial and industrial consumers. This inefficiency stems from the Jamaican tariff design not accounting for the inverse demand rule, which requires the Ramsey markup price over marginal costs to be inversely proportional to the elasticities of demand. He concludes that because prices under revenue cap regulation contravene this rule, revenue caps are welfare-decreasing compared to price caps.

Our analysis differs from Campbell (2018a) in that (i) we attribute our attention to a single market rather than analyze welfare effects across different market segments and (ii) only one representative year of a regulatory period (the base year) rather than each year influences the monopolist's allowed revenues. Consequently, in our model, the monopolist cannot discriminate prices between different market segments but across time periods in a single market. Interestingly, though, we find that in line with the results by Campbell (2018a), more inelastic demand leads to vaster inefficiencies of revenue caps. This result suggests some robustness of the identified connection between revenue cap performance and demand elasticity across regulatory variations. Finally, note that whereas Campbell (2018a) focuses on comparing price and revenue cap regulation regarding welfare implications, we propose a combination of both to induce the socially optimal outcome.

Since the ARegV treats the costs in base years as the basis for the upcoming regulatory period (Federal Ministry for Economic Affairs and Climate Action, 2021), we also contribute to the work of Pint (1992). She introduces a multi-period model with stochastic costs to investigate regulatory hearings' welfare effects. During the regulatory hearings, the regulator assesses the monopolist's costs to adjust the regulatory restrictions imposed upon the monopolist. Pint (1992) analyzes rate-of-return as well as price cap regulation. Regarding the timing of the hearings, she finds theoretical evidence that allowing the monopolist to concentrate pricing distortions in a single period (base year approach) gives the monopolist more latitude to manipulate the regulation than a system that evaluates the monopolist's average costs over multiple periods.

While Pint (1992) discusses the impact of a base year approach within rate-of-return and price cap regulation, there is a lack of discussion on how cost-based revenue caps perform concerning the pricing behavior of monopolists. We are closing this gap. Our results align with Pint (1992) as we also find that a base year approach may introduce incentives to create inefficiently high costs.

The remainder of this paper is as follows. Section 2 introduces our theoretical model, while section 3 studies this model under cost-based revenue regulation. In section 4, we present the hybrid regulatory approach as a means to prevent the inefficiencies exposed in section 3. Finally, we conclude this paper in section 5 by discussing our results.

2 Benchmark Model

We consider a simple model of a monopolist able to set any price $p \in [0, \tilde{p}]$. In line with Cowan (2007), the upper bound $\tilde{p}$ corresponds to the choke price above which the monopolist may not charge. Following Campbell (2018a), we model demand by a constant elasticity curve with elasticity below one, $q(p) = \alpha p^{-\epsilon}$ s.t. $\epsilon \in (0, 1)$.[3] One feature of such a demand curve is that revenue is strictly increasing in the price p. We opt for such a specification since it generally corresponds to a more favorable environment for revenue caps and, hence, a more conservative framework for studying their potential inefficiencies (see Comnes et al., 1995). Relaxing this assumption does not weaken our main arguments but rather seems to strengthen them.[4] The costs of supplying quantity q are given by the linear cost function $C(q) = F + c * q$.[5] Hence, the monopolist's profits for any price p correspond to $\pi(p) = q(p) * p - C(q(p))$. Without regulation, the monopolist charges the price that maximizes her profits. Since the revenue curve is a strictly increasing function of the price p, the optimal monopoly price corresponds to $\tilde{p}$. Throughout, we assume that the monopolist can operate profitably, $\pi(\tilde{p}) > 0$. The social welfare maximizing price equals marginal costs. However, we require the socially optimal price p^s to maximize social welfare under the constraint that the monopolist at least breaks even, $\pi(p^s) \geq 0$. Then, the socially optimal price p^s corresponds to the price that allows the monopolist to gain zero profits. Formally, $p^s = \arg_{p \in [0, \tilde{p}]}[\pi(p) = 0]$.

3 Revenue Cap Regulation

We continue by introducing a generalized version of the cost-based revenue cap employed by the ARegV. Each regulatory period consists of one base year and T non-base years. The monopolist's pricing strategy

[3] Empirical literature generally suggests that elasticity for energy demand is relatively small (see, among others, Wilhite et al., 1996; Bose and Shukla, 1999; Holtedahl and Joutz, 2004; Ziramba, 2008; Blázquez et al., 2013; De Vita et al., 2006; Campbell, 2018b; Ozaki, 2018; Freier and von Loessl, 2022).

[4] In particular, we can show that if the revenue curve is convex with a unique maximum, the monopolist always engages in inefficient behavior under the cost-based revenue cap and charges larger base year prices than non-base year prices. Hence, the current regulation never reaches the first best outcome. The underlying reason is that the revenue cap regulation introduces inefficiencies through two channels: (1) the base year approach as focused on in this paper and (2) the profitability quantity minimization for a given revenue as outlined by Crew and Kleindorfer (1986, 1996); Comnes et al. (1995); Lantz (2008). Our proposed hybrid regulation still constitutes an improvement to the revenue cap by itself and, under some conditions, may lead to the socially optimal outcome.

[5] As § 12 ARegV and Attachment 3 of the ARegV assume constant returns to scale for the linear optimization required for the benchmark regulation (Federal Ministry for Economic Affairs and Climate Action, 2021), we will also assume a constant relationship between input and output factors.

is a vector (p_b, p_n), where p_b indicates the price charged in the base year and p_n is the price in the non-base years. For each type of year $x \in \{b, n\}$, the monopolists revenue $p_x * q(p_x)$ may not exceed the revenue cap $\bar{R}_x$. The revenue cap $\bar{R}_x$ is endogenously determined by the monopolist's pricing strategy (p_b, p_n). In particular, it derives from her base year costs: $\bar{R}_x(p_b, p_n) = C(q(p_b))\ \forall x \in \{b, n\}$.

Throughout, we write the total profits over the base year and the T non-base years for any pricing strategy (p_b, p_n) as $\Pi(p_b, p_n) := \pi(p_b) + T\pi(p_n)$. Note that the regulation allows the monopolist to charge the socially optimal price in both periods. Under the given constraint, the monopolist maximizes $\Pi(p_b, p_n)$. Formally,

$$\max_{(p_b,p_n)\in[0,\tilde{p}]^2} [\Pi(p_b, p_n)] \text{ s.t. } p_x * q(p_x) \leq C(q(p_b))\ \forall x \in \{b, n\}. \tag{1}$$

Consider the pricing strategy $(p_b, p_n) = (p^s, p^s)$ as a starting point. The monopolist cannot increase p_b as this would violate the revenue cap in the base year. However, she may decrease p_b to some $\hat{p}_b$, which increases her base year costs above revenue, yielding a loss, $C(q(\hat{p}_b)) > \hat{p}_b q(\hat{p}_b)$. However, the cost increase enables the monopolist to increase the non-base year price to $\hat{p}_n > p^s$. Consequently, in the T non-base years, the monopolist generates revenue above costs and thus positive profits, $C(q(\hat{p}_n)) < \hat{p}_n q(\hat{p}_n)$. The choke price restricts the maximum profit that the monopolist can generate in the non-base years. Hence, taken to an extreme, the monopolist can decrease p_b to the price that generates costs at the monopoly revenue level, $C(q(p_b)) = \tilde{p} * q(\tilde{p})$, to charge monopoly non-base year prices, $p_n = \tilde{p}$, and generate the respective profits in the T non-base years, $[\tilde{p}q(\tilde{p}) - C(q(\tilde{p}))] * T > 0$. Depending on which strategy yields greater profits, the monopolist either charges the socially optimal price in both periods or decreases prices in the base year to reap monopoly profits in the non-base years. Figure 1 presents a graphical illustration of the above. The following proposition summarizes the result formally.

Proposition 3.1 (Optimal Pricing Strategy Under Revenue Cap)**.** *Let* $(p_b^r, p_n^r) \in [0, \tilde{p}]^2$ *be the solution of maximization 1 and* $\tilde{p}^0 := (\frac{\alpha * \tilde{p}^{1-\epsilon} - F}{\alpha c})^{-\frac{1}{\epsilon}} > 0$.

1. $(T-1)\tilde{p}q(\tilde{p}) + \tilde{p}^0 q(\tilde{p}^0) > TC(\tilde{p}) \Rightarrow (p_b^r, p_n^r) = (\tilde{p}^0, \tilde{p})$.

2. $(T-1)\tilde{p}q(\tilde{p}) + \tilde{p}^0 q(\tilde{p}^0) \leq TC(\tilde{p}) \Rightarrow (p_b^r, p_n^r) = (p^s, p^s)$.

Proof: see appendix A.1.

We can rearrange the proposition to state that if in the absence of regulation, the monopolist faces a sufficiently large profit margin, $\frac{\tilde{p}*q(\tilde{p}) - C(\tilde{p})}{\tilde{p}*q(\tilde{p})} > \frac{1}{T}$, the cost-based revenue cap incentivizes the monopolist to employ pricing strategy $(p_b^r, p_n^r) = (\tilde{p}^0, \tilde{p})$, yielding a socially inefficient outcome. Rearranging the condition yields $\frac{\tilde{p}*q(\tilde{p}) - C(\tilde{p})}{\tilde{p}*q(\tilde{p})} > \frac{1}{T} \Leftrightarrow 1 - \frac{F}{\alpha \tilde{p}^{1-\epsilon}} - \frac{c}{\tilde{p}} > \frac{1}{T}$. Hence, the monopolist likely engages in socially inefficient pricing if normalized fixed costs $\frac{F}{\alpha}$, marginal costs c and price elasticities ϵ are small, and the choke price $\tilde{p}$, as well as the number of non-base years T are large. Given the small observed elasticity regarding energy prices, large consumer dependency on energy supply, and the relatively large number of non-base years, the

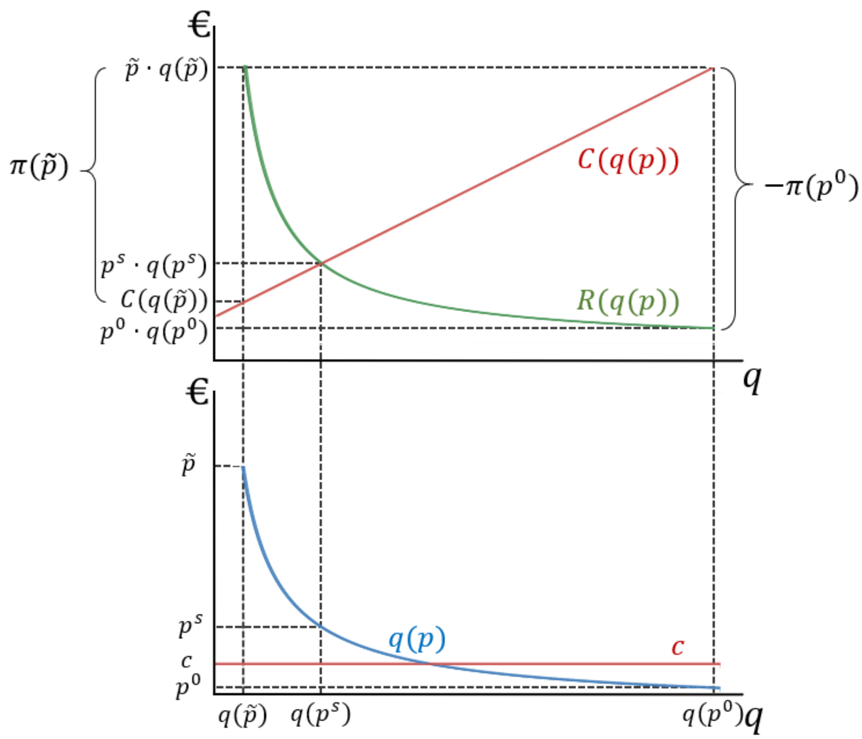

Figure 1: pricing strategy under revenue cap

above seems to be a realistic threat. For the remainder of the analysis, we suppose that the above holds and the revenue cap does not induce the first best outcome.

We can analyze welfare implications for society if the monopolist does not engage in the socially optimal pricing strategy. Under the revenue cap constraint, as compared to the unrestricted monopoly case, the monopolist gains less profits, and consumers obtain a greater surplus. This holds because, although the situation mimics the unrestricted monopoly case in the non-base years, the base year exhibits decreased prices and increased quantities. The total effect on welfare is ambiguous.

Proposition 3.2. *The revenue cap is welfare enhancing as compared to the unregulated monopoly if (1) $\epsilon \geq \frac{1}{2}$ or (2) $\tilde{p} \in [p^s, \tilde{\tau})$ for some critical $\tilde{\tau} \in \mathbb{R}$, where $\tilde{\tau}$ increases in c, $\frac{F}{\alpha}$, and ϵ.*

Proof: see appendix A.1.

The above proposition states that the revenue cap is welfare enhancing if the marginal costs c, normalized fixed costs $\frac{F}{\alpha}$, or elasticity ϵ is large, or the monopoly price $\tilde{p}$ is small. Larger cost parameters c and $\frac{F}{\alpha}$ facilitate cost inflation in the base year, ensuring that the price decrease and quantity increase in the base year is relatively small. Similarly, if the choke price $\tilde{p}$ is relatively small, so is the monopoly revenue in the unregulated monopoly case. To reap these revenues in the non-base years, the monopolist requires relatively small costs in the base year. Hence, she again engages in only a moderate price decrease and corresponding quantity increase in the base year. Such moderate price decreases and corresponding quantity increases ensure the absence of drastic welfare losses. Lastly, consider elasticity ϵ. To reap maximum profits in the non-base years, the monopolist must produce the quantity that generates the equivalent costs in the base year. As elasticity ϵ decreases, so does the base year price that yields this quantity being demanded. Since a smaller price indicates a smaller marginal utility of consumption, a smaller elasticity ϵ corresponds to lower consumption benefits.

4 Hybrid Regulation

The previous section illustrates incentives set by the revenue cap to engage in inefficient pricing. We now present an extension of the regulation addressing this issue: namely, a price cap $\bar{p} \in [p^s, \tilde{p})$, above which the monopoly may not charge. The updated monopolist's maximization problem under this hybrid regulation looks as follows:

$$\max_{(p_b,p_n)\in[0,\bar{p}]^2} [\Pi(p_b, p_n)] \text{ s.t. } p_x * q(p_x) \leq C(q(p_b)) \ \forall x \in \{b, n\}. \tag{2}$$

As in the previous section, there are two sensible strategies for the monopolist. First, she can charge the break-even price p^s in each period. Alternatively, she may bump up costs in the base year to charge the largest possible price in the non-base years, now given by the price cap $\bar{p}$. Formally, the following lemma captures the monopolist's optimal pricing strategy under the hybrid regulation.

Lemma 4.1 (Optimal Pricing Strategy Under Hybrid Regulation). *Let $(p_b^h, p_n^h) \in [0, \bar{p}]^2$ be the solution of maximization 2 and $\bar{p}^0 := (\frac{\alpha * \bar{p}^{1-\epsilon} - F}{\alpha c})^{-\frac{1}{\epsilon}} > 0$.*

1. $(T-1)\bar{p}q(\tilde{p}) + \bar{p}^0 q(\bar{p}^0) > TC(\bar{p}) \Rightarrow (p_b^h, p_n^h) = (\bar{p}^0, \bar{p})$.
2. $(T-1)\bar{p}q(\tilde{p}) + \bar{p}^0 q(\bar{p}^0) \leq TC(\bar{p}) \Rightarrow (p_b^h, p_n^h) = (p^s, p^s)$.

Proof: mimics that of proposition 3.1 with $\bar{p}$ instead of $\tilde{p}$.

Lemma 4.1 resembles proposition 3.1, the only difference being the maximum attainable price. Note that under the revenue cap alone, $p_n^r = \bar{p} \in (p^s, \tilde{p})$ is feasible but never optimal. Hence, if charging $\tilde{p}$ in the non-base years is optimal, it must yield greater profits than charging $\bar{p}$ in the non-base years. Hence, if it is optimal to charge $\bar{p}$ under the hybrid regulation, it must be optimal to charge $\tilde{p}$ under the revenue cap. This indicates that, unlike revenue cap regulation by itself, the hybrid regulation with $\bar{p}$ incentivizes the monopolist to charge the socially optimal price. We can show that regarding social welfare, the hybrid regulation always performs at least as well as the revenue cap regulation.

Proposition 4.1 (Hybrid Regulation vs. Revenue Cap). *(p_b^h, p_n^h) is always better (or at least as good) as (p_b^r, p_n^r) from a social welfare perspective.*

Proof: see appendix A.1.

If the revenue cap alone yields the socially optimal outcome, then combining it with the price cap does not alter the situation, and the hybrid regulation also yields the socially optimal outcome. Moreover, if the hybrid regulation leads to the socially optimal outcome, it must be at least as good as the revenue cap regulation. Hence, we must argue why the hybrid regulation outperforms the revenue cap if neither regulation induces the socially optimal outcome. Note that for neither regulation to reach the socially optimal outcome, it must be that both pricing strategies $(\bar{p}^0, \bar{p})$ and $(\tilde{p}^0, \tilde{p})$ yield positive profits. Moreover, since $(\bar{p}^0, \bar{p})$ is attainable for the monopolist under the revenue cap, but she chooses $(\tilde{p}^0, \tilde{p})$, the second pricing strategy must yield greater total profits, $\Pi(\tilde{p}^0, \tilde{p}) > \Pi(\bar{p}^0, \bar{p}) \geq 0$. Higher monopoly profits must come at social costs, leading to an overall loss in social welfare. Moreover, social welfare under the break-even constraint is maximized for the pricing strategy (p^s, p^s). As the pricing strategy under the hybrid regulation seems closer to the socially optimal outcome than the pricing strategy under the revenue cap, we would expect welfare to increase through the hybrid regulation.

We briefly discuss how the hybrid regulation performs against the price cap regulation. Therefore, note that when restricted solely by the price cap $\bar{p}$, the monopolist maximizes profits by charging this cap in each period. Hence, her optimal pricing strategy is $(\bar{p}, \bar{p})$.

Proposition 4.2 (Hybrid Regulation vs. Price Cap)**.** *(p_b^h, p_n^h) generates at least as much social welfare as $(\bar{p}, \bar{p})$ if (1) (p_b^r, p_n^r) generates at least as much social welfare as $(\tilde{p}, \tilde{p})$ or (2) $\bar{p} \in [p^s, \bar{\tau})$ for some critical $\bar{\tau} \in \mathbb{R}$.*

Proof: see appendix A.1.

The proposition states that the hybrid regulation generates more social welfare than the price cap if either (1) the revenue cap generates more than no regulation or (2) the price cap is sufficiently close to the socially optimal price p^s. Hence, if there is a strong argument for either of the two individual regulations (price cap or revenue cap), then the hybrid regulation is likely superior to the price cap. The result of proposition 4.2 indicates that the hybrid regulation is sometimes welfare superior to the price cap alone, even if the revenue cap is welfare inferior compared to no regulation. In particular, this holds if proposition 3.2 does not apply but $\bar{p}$ is sufficiently close to p^s and, thus, proposition 4.2 holds. Hence, the hybrid regulation is more likely welfare superior to any price cap $\bar{p} \in (p^s, \tilde{p})$, the closer the respective price cap is to the social optimum p^s and, hence, the greater the welfare gains from employing this particular price cap as compared to an absence of regulation. Finally, we evaluate the efficiency of the hybrid regulation in absolute terms.

Proposition 4.3 (First Best Outcome under Hybrid Regulation)**.** *If $T < \frac{1-\epsilon}{\epsilon} \frac{p^s}{c}$, then $\exists \delta > 0$ s.t. $(p_b^h, p_n^h) = (p^s, p^s)$ $\forall \bar{p} \in [p^s, p^s + \delta)$.*

Proof: see appendix A.1.

The proposition states that, under the given condition, a connected set of price caps exists so that the hybrid regulation reaches the first best outcome for each element in this set. The underlying reason is that charging the socially optimal price in both periods is a local maximum for the monopolist. By letting the price cap be sufficiently close to p^s, the monopolist's feasible strategies are on the domain on which (p^s, p^s) maximizes her profits. Hence, the hybrid regulation induces the socially optimal outcome.

Note that the condition of proposition 4.3 is equivalent to $T < -\frac{\mathrm{d}p^0(p)}{\mathrm{d}p}|_{p=p^s}$, where $p^0(p)$ corresponds to the base year price that generates sufficiently high costs to enable charging the non-base year price p. Based on this alternative specification, we can intuitively interpret our results. First, $(p_b, p_n) = (p^s, p^s)$ is more likely to be a local maximum of monopoly profits if there are fewer non-base year periods T since it becomes less profitable to incur a loss in the base year to reap profits in the non-base years. Second, social optimal pricing $(p_b, p_n) = (p^s, p^s)$ is likely locally optimal if a marginal increase in p_n requires a drastic decrease in p_b since such a decrease directly lowers the base years revenue and thus, total profits. From proposition 4.3 it is apparent that $-\frac{\mathrm{d}p^0(p)}{\mathrm{d}p}|_{p=p^s} = \frac{1-\epsilon}{\epsilon}\frac{p^s}{c}$ is relatively large if elasticity ϵ is small. If ϵ is small, demand is non-responsive to price changes, which requires a large decrease in the base year price to increase demand, the base year's costs, and, hence, the revenue cap. This large price decrease drastically lowers revenue and, thus, profits in the base year.

Moreover, we can investigate the role of marginal costs c and normalized fixed costs $\frac{F}{\alpha}$ through their influence on $\frac{p^s}{c}$. First, note that for fixed marginal costs c, a greater break-even price p^s yields a smaller change in quantity resulting from a price change at p^s in absolute terms. This becomes apparent by noting that the revenue curve in figure 1 becomes steeper, the smaller q and, hence, the larger p is. By similar reasoning as above, larger p^s implies that the revenue decrease in the base year associated with a marginal increase in the non-base year's price is more drastic. An increase in the normalized fixed costs $\frac{F}{\alpha}$ increases the break-even price p^s and favors the hybrid regulation to yield the socially optimal outcome. The effect of c is not as clear-cut. Although an increase in c also increases p^s, favoring (p^s, p^s) to be a local maximum through the above-described mechanism, it also has a countervailing effect: For larger marginal costs c, the required increase in the base year's quantity to reach a certain cost level decreases. Hence, a marginal increase in the non-base year's price requires a smaller decrease in the base year's price. We can formally show that this second effect generally dominates the first one, so the hybrid regulation is more likely to yield the socially optimal outcome if marginal costs are small.

5 Discussion

We have shown that under the cost-based revenue cap and some conditions that plausibly hold for DSOs, a monopolist has incentives to engage in inefficient pricing by increasing costs in the base year to reap profits in the non-base years. By combining the revenue cap with a price cap, social welfare unambiguously increases and, again, under some conditions that plausibly hold for DSOs, society may even reach the socially optimal outcome. If the latter applies, then an admirable property of the hybrid regulation is that it holds for a connected set of price caps. Hence, regulatory authorities do not need exact market and firm structure information to regulate the monopolist efficiently. Consequently, some price caps that would not induce the socially optimal behavior by themselves do induce such an outcome in combination with the cost-based revenue cap. In that sense, the hybrid regulation dominates the cost-based revenue cap and the price cap by themselves in terms of incentivizing efficient pricing by the monopolist.

We can extend our analysis to show that we obtain the same results if we employ a cap on price differences or ratios rather than actual prices. This may be the more feasible regulation in practice as the DSOs differ in cost and market structures. Introducing a universal price cap would not account for such differences, whereas DSO-specific price caps presumably require extensive information collection and organizational costs.

An alternative for eliminating the incentives for inefficient pricing is to abandon the differentiation of base and non-base years. Hence, each year's revenue cap coincides with the previous year's costs. However, doing so could raise other issues. First, it likely reduces incentives to engage in cost-reducing behavior as the resulting gap between costs and maximum revenue only pays off in one year rather than the whole regulatory period. Second, the yardstick competition mechanisms of the ARegV compare the cost efficiencies of DSOs. Inefficiently operating DSOs are sanctioned through a reduction of their revenue cap below their actual costs, creating some quasi-competition to incentivize cost-reducing measures. However, collecting and processing the relevant information is costly and likely unfeasible each year.

Note that although our analysis focuses on the particular case of cost-based revenue caps (as employed in the ARegV), our insights carry over to other forms of endogenous revenue caps employing a base year approach. Therefore, the results, as well as our proposed solution, are of more general appeal.

To further deepen our theoretical understanding of the subject matter, future research should investigate how the pricing incentives interact with the cost-reduction incentives of the cost-based revenue cap in a dynamic setting. Moreover, empirical analyses studying current equilibrium conditions may prove helpful in establishing whether a hybrid regulation may yield the first best outcome or whether an informal price-difference cap is already in place.

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A Appendix

A.1 Proofs

Before proceeding with the formal proofs, we introduce some definitions that will prove helpful later on. Moreover, throughout the analysis, we write $f = F/\alpha$.

Definition A.1. For any two prices $p_1, p_2 \in \mathbb{R}_{\geq 0}$, the difference in social welfare in one period from these prices is $\Delta wf(p_1, p_2) := \int_{q(p_1)}^{q(p_2)} (p(q(p)) - c)dq(p) = \int_{q(p_1)}^{q(p_2)} ((\frac{q(p)}{\alpha})^{\frac{1}{-\epsilon}} - c)dq(p) = [\frac{\epsilon}{(\epsilon-1)}\alpha^{\frac{1}{\epsilon}} q(p)^{\frac{\epsilon-1}{\epsilon}} - cq]_{q(p_2)}^{q(p_1)} = \alpha * [\frac{\epsilon}{(\epsilon-1)} p^{1-\epsilon} - cp^{-\epsilon}]_{p_2}^{p_1}$. Let $\Delta wf = \frac{wf}{\alpha}$.

Definition A.2. Social welfare for any pricing strategy (p_b, p_n) is defined relative to the unregulated monopoly: $WF(p_b, p_n) = \Delta wf(p_b, \tilde{p}) + T\Delta wf(p_n, \tilde{p})$.

Definition A.3. For any $p \in \mathbb{R}_{\geq 0}$, let $p^0(p)$ be s.t. $C(q(p^0)) = q(p) * p$. Hence, $p^0(p) = (\frac{p^{1-\epsilon} - f}{c})^{-\frac{1}{\epsilon}}$.

Lemma A.1.

If $\epsilon \geq 0,5$ *, then* $\nexists \hat{p} > p^s$ *s.t* $\Delta wf'(p^0(\hat{p}), \hat{p}) = 0$.

If $\epsilon < 0,5$ *, then* $! \exists \hat{p} > p^s$ *s.t* $\Delta wf'(p^0(\hat{p}), \hat{p}) = 0$.

Proof. We start by deriving $\Delta wf'(p^0(p), p) = \alpha p^{-\epsilon}[(1-\epsilon)(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1-2\epsilon) - \epsilon\frac{c}{p}]$. Let $\Delta \tilde{wf}'(p^0(p), p) = (1-\epsilon)(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1-2\epsilon) - \epsilon\frac{c}{p}$. $\Delta wf'(p^0(p), p) > 0 \Leftrightarrow \Delta \tilde{wf}'(p^0(p), p) > 0$ and $\Delta wf'(p^0(p), p) < 0 \Leftrightarrow \Delta \tilde{wf}'(p^0(p), p) < 0$.

First, suppose $\epsilon \geq \frac{1}{2}$. We can write $\Delta \tilde{wf}'(p^0(p), p) = \epsilon(1 - \frac{c}{p}) - (1-\epsilon)(1 - \frac{c^{\frac{1-\epsilon}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}}})$. Note that $\epsilon \geq \frac{1}{2}$ implies (1) $\epsilon > 1 - \epsilon$ and (2) $\frac{c}{p} \leq \frac{c^{\frac{1-\epsilon}{\epsilon}}}{p^{\frac{1-\epsilon}{\epsilon}}} < \frac{c^{\frac{1-\epsilon}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}}}$ $\forall f > 0$. Hence, $\Delta \tilde{wf}'(p^0(p), p) > 0$, which proves the first statement of the lemma.

Second, suppose $\epsilon < \frac{1}{2}$. Note that $\lim_{p \to \infty}(\Delta \tilde{wf}'(p^0(p), p) = 2\epsilon - 1 < 0$. To prove the statement, it suffices to show that $\frac{d\tilde{wf}'(p^0(p), p)}{dp} = \tilde{wf}''(p^0(p), p) < 0$. We can derive $\tilde{wf}''(p^0(p), p) = \frac{1}{c\epsilon}(\epsilon^2\frac{c^2}{p^2} - (1-\epsilon)^2\frac{c^{\frac{1}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}+1}p^{\epsilon}})$. Note that $\epsilon^2 < (1-\epsilon)^2$. Hence, $\tilde{wf}''(p^0(p), p) < 0$ holds if $\frac{c^2}{p^2} < \frac{c^{\frac{1}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}+1}p^{\epsilon}}$. Suppose $f = 0$. $\frac{c^{\frac{1}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}+1}p^{\epsilon}} = \frac{c^{\frac{1}{\epsilon}}}{p^{\frac{1}{\epsilon}}}$. For $\epsilon < \frac{1}{2}$, $\frac{1}{\epsilon} > 2$ and, hence, $\frac{c^2}{p^2} < \frac{c^{\frac{1}{\epsilon}}}{p^{\frac{1}{\epsilon}}} < \frac{c^{\frac{1}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}+1}}$ $\forall f > 0$. Thus, $\tilde{wf}''(p^0(p), p) < 0$, which proves the second statement of the lemma. □

Proof of Proposition 3.1:

Proof. To prove the proposition, we divide Π by α and maximize this function using Lagrange. The Lagrange function for the adjusted maximization problem is $\mathcal{L} = p_b^{1-\epsilon} - f - cp_b^{-\epsilon} + T(p_n^{1-\epsilon} - f - cp_n^{-\epsilon}) - \mu_1(p_b^{1-\epsilon} - f - cp_b^{-\epsilon}) - \mu_2(p_n^{1-\epsilon} - f - cp_b^{-\epsilon}) - \mu_3(p_n - \tilde{p})$, yielding the conditions: (i.) $\frac{\partial \mathcal{L}}{\partial p_b} = (1-\epsilon)p_b^{-\epsilon}(1-\mu_1) + \epsilon c p_b^{-1-\epsilon}(1 - \mu_1 - \mu_2) = 0$, (ii.) $\frac{\partial \mathcal{L}}{\partial p_n} = ((1-\epsilon)p_n^{-\epsilon} + \epsilon c p_n^{-1-\epsilon})(1-\mu_2) - \mu_3 = 0$, (iii.) $\mu_1(p_b^{1-\epsilon} - f - cp_b^{-\epsilon}) = 0$, (iv.) $\mu_2(p_n^{1-\epsilon} - f - cp_b^{-\epsilon}) = 0$, (v.) $\mu_3(p_n - \tilde{p}) = 0$. Note that for $\mu_1 = \mu_2 = \mu_3 = 0$ and all strictly positive prices, (i.) and (ii.) are greater than zero. If $\mu_2 = 0 \wedge (p_n^{1-\epsilon} < f + cp_b^{-\epsilon})$, then $\mu_3 > 0$. Otherwise, (ii.) cannot be true. Hence, $p_n = \tilde{p}$. Moreover, $\mu_1 = 1$ for (i.) to hold. Hence, $p_b = p^s$. But then, $p_n^{1-\epsilon} < f + cp_b^{-\epsilon}$ cannot be true since $\tilde{p}^{1-\epsilon} > f + cp^{s-\epsilon}$. Hence, throughout holds $\mu_2 \neq 0$, implying $p_n^{1-\epsilon} = f + cp_b^{-\epsilon}$. Next, suppose $\mu_3 \neq 0 \wedge \tilde{p} = p_n$. $\mu_3 \neq 0 \wedge \tilde{p} = p_n \Rightarrow p_b$ solves $\tilde{p}^{1-\epsilon} = f + cp_b^{-\epsilon}$. Alternatively, $\tilde{p} \neq p_n \Rightarrow \mu_3 = 0$. For (ii.) to be zero, it must be that $\mu_2 = 1$. $\mu_2 = 1 \Rightarrow \mu_1 \neq 0$. Otherwise, (1) is not zero. Hence, $p_b^{1-\epsilon} = f + cp_b^{-\epsilon}$, implying $p_b = p^s$. Consequently, $p_n = p_s$. We have determined two possibly optimal strategies: 1. (p^s, p^s) and 2. $(\tilde{p}^0, \tilde{p})$. The strategy yielding greater profits is the global maximum. This is captured by (1) and (2) of the proposition. □

Proof of Proposition 3.2:

Proof. First, suppose $(p_b^r, p_n^r) = (p^s, p^s)$. The revenue cap yields the first best outcome and, thus, must enhance welfare. For the remainder, suppose $(p_b^r, p_n^r) = (\tilde{p}^0, \tilde{p})$. The revenue cap is welfare enhancing if $\Delta wf(\tilde{p}^0, \tilde{p}) > 0$ since the revenue cap then generates greater social welfare in the base year. In the non-base years, it generates the same social welfare.

We start with the (2) of the proposition. Therefore, consider $\epsilon < \frac{1}{2}$, since otherwise (1) of the proposition implies (2). Consider $\Delta wf'(p^0(p), p) = \alpha p^{-\epsilon}[(1-\epsilon)(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1-2\epsilon) - \epsilon\frac{c}{p}]$.

Let p^s be a function of f: $p^s(f)$. Note that $p^s(0) = c$, $p^s(f) > c \forall f > 0$, and ${p^s}^{1-\epsilon} - f = p^{s-\epsilon} * c$. We can then easily show that $\Delta wf'(p^0(p), p) = 0$ for $p^s(f)$ and $f = 0$. Moreover, $wf'(p^0(p), p) > 0$ for $p = p^s \Leftrightarrow [(1-\epsilon)(\frac{{p^s}^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1-2\epsilon) - \epsilon\frac{c}{p^s}] > 0 \Leftrightarrow [(1-\epsilon)\frac{p^s}{c} - (1-2\epsilon) - \epsilon\frac{c}{p^s}] > 0$. This expression increases in p^s and p^s increases in f. Hence, $f > 0$ implies $\Delta wf'(p^0(p), p) > 0$ at $p = p^s$. Since $\Delta wf(p^0(p^s), p^s) = \Delta wf(p^s, p^s) = 0$, $\exists \delta > 0$ s.t. $\Delta wf(p^0(\hat{p}), \hat{p}) > 0 \; \forall \hat{p} \in (p^s, p^s + \delta)$. Moreover, from lemma A.1, $\lim_{p\to\infty} \Delta wf(p^0(\hat{p}), \hat{p}) < 0$, and $\Delta wf'(p^0(p^s), p^s) > 0$ follows that: $! \exists \tau > p^s$ s.t. $\Delta wf(p^0(\check{p}), \check{p}) < 0$ if and only if $\check{p} > \tau$. Consider $x \in \{f, c, \epsilon\}$. Recall that $\Delta wf'(p^0(p), p) = \alpha p^{-\epsilon}[(1-\epsilon)(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1-2\epsilon) - \epsilon\frac{c}{p}] > 0 \Leftrightarrow (1-\epsilon)(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1 - 2\epsilon) - \epsilon\frac{c}{p} = \epsilon(1 - \frac{c}{p}) - (1-\epsilon)(1 - \frac{c^{\frac{1-\epsilon}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}}}) > 0$. Hence, if at any p s.t. $\Delta wf(p^0(p), p) \geq 0$ the term $\epsilon(1 - \frac{c}{p}) - (1-\epsilon)(1 - \frac{c^{\frac{1-\epsilon}{\epsilon}}}{(p^{1-\epsilon}-f)^{\frac{1}{\epsilon}}})$ increases in x, then it must hold that τ increases in x. By inspecting the term more closely, we can see it holds for c, f, and ϵ, proving the second statement of the proposition.

Next, we consider the first condition of the proposition. From A.1, we know if $\epsilon \geq 0,5$, then $\nexists \hat{p} > p^s$ s.t. $wf'(\hat{p}, \hat{p}) = 0$. From the first part of the proof, we know $f > 0$ implies $\Delta wf'(p^s, p^s) > 0$. In conjunction, it follows that $\Delta wf'(p^0(p), p) > 0$ for all $p > p^s$, and, thus, $\Delta wf(p^0(p), p) > 0$ for all $p > p^s$. Hence, $\Delta wf(\tilde{p}^0), \tilde{p}) > 0$. □

Proof of Proposition 4.1:

Proof. Suppose $(p_b^r, p_n^r) = (p^s, p^s)$. Hence, $(p_b^h, p_n^h) = (p^s, p^s)$. Otherwise $\Pi(p_b^h, p_n^h) > 0 = \Pi(p_b^r, p_n^r)$, but (p_b^h, p_n^h) is attainable at the revenue cap. Hence, we would reach a contradiction. $(p_b^r, p_n^r) = (p^s, p^s) = (p_b^h, p_n^h)$ implies that welfare is equal. Next, suppose $(p_b^r, p_n^r) \neq (p^s, p^s)$ and $(p_b^h, p_n^h) = (p^s, p^s)$. Recall that (p^s, p^s) maximizes social welfare under the constraint that the monopolist at least breaks even. Since $(p_b^r, p_n^r) \neq (p^s, p^s) \Rightarrow \Pi(p_b^r, p_n^r) > 0$, (p_b^r, p_n^r) cannot generate more social welfare than (p_b^h, p_n^h). Lastly, assume

$(p_b^r, p_n^r) \neq (p^s, p^s) \neq (p_b^h, p_n^h)$. $(p^s, p^s) \neq (p_b^h, p_n^h) \Rightarrow \Pi(p_b^r, p_n^r) > 0$. Consider any pricing strategy $(p^0(p), p)$ s.t. $p \geq p^s$. From the proof of proposition 3.1, we know that there are two potential maxima of $\Pi(p^0(p), p)$ that lie at the boundaries of attainable prices. Hence, there is no local maximum $p^l \in (p^s, \tilde{p})$ of $\Pi(p^0(p), p)$, implying: $\Pi(p^0(p), p) > 0 \Rightarrow \Pi'(p^0(p), p) \geq 0 \Rightarrow \Pi(p^0(\hat{p}), \hat{p}) > 0 \, \forall \hat{p} > p$. Since $p_b^h < p_b^r$ and $\Pi(p_b^r, p_n^r) > 0$, to proof the proposition it suffices to show that for all $p > p^s$, $\Pi(p^0(p), p) > 0 \Rightarrow \frac{\mathrm{d}WF(p^0(p),p)}{\mathrm{d}p} < 0$.

Consider $\frac{\mathrm{d}WF(p^0(p),p)}{\mathrm{d}p} = WF'(p^0(p), p) = p^{-\epsilon}((\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} - 1 + \epsilon - \epsilon T + \epsilon T\frac{c}{p})$. Since (1) $(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} - 1 + \epsilon - \epsilon T + \epsilon T\frac{c}{p}$ strictly decreases $\forall p > 0$, (2) $\lim_{p\to\infty}((\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} + \epsilon T\frac{c}{p}) \leq 0$, and (3) $-1 + \epsilon - \epsilon T < 0$, $\exists! \, p^* \in (0, \infty)$ s.t. $WF'(p^0(p^*), p^*) = 0$. For all $p > p^*$, $WF'(p^0(p), p) < 0$. Note, $(\frac{x^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} + \epsilon T\frac{c}{x} < (\frac{y^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} + \epsilon T\frac{c}{y} \Leftrightarrow x > y$. Hence, $p^s \geq p^* \Leftrightarrow (\frac{{p^s}^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} + \epsilon T\frac{c}{p^s} \leq (\frac{{p^*}^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} + \epsilon T\frac{c}{p^*} = 1 - \epsilon + \epsilon T \Leftrightarrow p^s\frac{1-\epsilon}{c} + \epsilon T\frac{c}{p^s} \leq 1 - \epsilon + \epsilon T \Leftrightarrow \frac{1-\epsilon}{\epsilon}\frac{p^s}{c} \leq T$. Thus, $\frac{1-\epsilon}{\epsilon}\frac{p^s}{c} \leq T \Rightarrow p^s \geq p^* \Rightarrow (WF'(p^0(p), p) < 0 \forall p > p^* \Rightarrow WF'(p^0(p), p) < 0 \forall p > p^s)$.

Alternatively, suppose $\frac{1-\epsilon}{\epsilon}\frac{p^s}{c} > T$. Hence, $\frac{1-\epsilon}{\epsilon}\frac{p}{c} > T \, \forall p > p^s$. Consider $\Pi'(p^0(p), p) = p^{-\epsilon}(\frac{\epsilon-1}{\epsilon}(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} + (1-\epsilon)(T-1) + \epsilon T\frac{c}{p}) > 0 \Leftrightarrow (\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} < \frac{\epsilon}{1-\epsilon}[(1-\epsilon)(T-1) + \epsilon T\frac{c}{p}]$. It follows that $WF'(p^0(p), p) = p^{-\epsilon}((\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} - 1 + \epsilon - \epsilon T + \epsilon T\frac{c}{p}) < 0 \Leftrightarrow (\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1-\epsilon}{c} - 1 + \epsilon - \epsilon T + \epsilon T\frac{c}{p} < 0$ is true if $\frac{\epsilon}{1-\epsilon}[(1-\epsilon)(T-1) + \epsilon T\frac{c}{p}] - 1 + \epsilon - \epsilon T + \epsilon T\frac{c}{p} < 0$. $\frac{\epsilon}{1-\epsilon}[(1-\epsilon)(T-1) + \epsilon T\frac{c}{p}] - 1 + \epsilon - \epsilon T + \epsilon T\frac{c}{p} < 0 \Leftrightarrow T < \frac{1-\epsilon}{\epsilon}\frac{p}{c}$. Thus, $\Pi'(p^0(p), p) > 0 \Rightarrow WF'(p^0(p), p) < 0 \, \forall p > p^s$. □

Proof of Proposition 4.2:

Proof. Suppose $(p_b^h, p_n^h) = (p^s, p^s)$. Hence, the pricing strategy maximizes welfare, given that the monopolist breaks even. Both statements must evidently be true. Suppose $(p_b^h, p_n^h) \neq (p^s, p^s) \Rightarrow (\bar{p}^0, \bar{p})$. Differences in (p_b^h, p_n^h) and $(\bar{p}, \bar{p})$ only arise from $\bar{p}^0$ and $\bar{p}$. Hence, we need to investigate $\Delta wf(p^0(p), p)$.

We first investigate the second statement of the proposition. Recall from the proof of proposition 3.2 that $f > 0$ implies $\Delta wf'(p^0(p), p) > 0$ at $p = p^s$. Since $\Delta wf_{p^s} = \Delta wf(p^0(p^s), p^s) = \Delta wf(p^s, p^s) = 0$, $\exists \delta > 0$ s.t. $\Delta wf(p^0(\hat{p}), \hat{p}) > 0 \, \forall \hat{p} \in (p^s, p^s + \delta)$.

We continue with the first statement of the proposition. As above, we must compare the welfare resulting from $\bar{p}$ with $\bar{p}^0$. From the above, we know that at $p = p^s$, $\Delta wf'(p^0(p), p) > 0$ and $\Delta wf(p^0(p), p) = 0$. Moreover, $\Delta wf'(p^0(p), p) = \alpha p^{-\epsilon}[(1-\epsilon)(\frac{p^{1-\epsilon}-f}{c})^{-\frac{1}{\epsilon}}\frac{1}{c} - (1-2\epsilon) - \epsilon\frac{c}{p}]$. From A.1 we know that if $\epsilon \geq 0{,}5$, then $\nexists \hat{p} > p^s$ s.t $\Delta wf'(p^0(\hat{p}), \hat{p}) = 0$. Since $\Delta wf'(p^0(p^s), p^s) > 0$ and $\Delta wf(p^0(p^s), p^s) = 0$, this implies $\Delta wf'(p^0(\hat{p}), \hat{p}) > 0 \, \forall \hat{p} > p^s$ and, consequently, $\Delta wf(p^0(\hat{p}), \hat{p}) > 0 \, \forall \hat{p} > p^s$. Alternatively, if $\epsilon < 0{,}5$, then $\exists! \, \hat{p} > p^s$ s.t $\Delta wf'(p^0(\hat{p}), \hat{p}) = 0$. Since for $\epsilon < 0{,}5$, $\lim_{p\to\infty}(wf(p^0(p), p)) < 0$, $\exists! \, \hat{p} > p^s$ s.t. $\Delta wf(p^0(\hat{p}), \hat{p}) = 0$. Hence, $\Delta wf(p^0(\hat{p}), \hat{p}) > 0$ for $\hat{p}$ implies that $\Delta wf(p^0(\check{p}), \check{p}) > 0$ for all $\check{p} \in (p^s, \hat{p}]$. Since $\tilde{p} > \bar{p}$, it holds that for all $\epsilon \in (0, 1)$: $\Delta wf(p^0(\tilde{p}), \tilde{p}) > 0 \Rightarrow \Delta wf(p^0(\hat{p}), \hat{p}) > 0$. □

Proof of Proposition 4.3:

Proof. Notice that $\frac{\mathrm{d}p^0(p)}{\mathrm{d}p} = -\frac{1}{\epsilon}(\frac{p^{1-\epsilon}-f}{c})^{\frac{-1-\epsilon}{\epsilon}}(1-\epsilon)\frac{p^{-\epsilon}}{c}$. Recall that $p^0(p^s) = p^s$ and $\Pi(p^0(p^s), p^s) = 0$. The proposition is true if $\frac{\mathrm{d}\Pi(p^0(p),p)}{\mathrm{d}p}|_{p=p^s} < 0$. Verbally, at the socially optimal price p^s, a decrease in the base year price and a corresponding increase in the non-base year price decrease overall profits. $\Pi(p^0(p), p) = p^0(p)^{1-\epsilon} - cp^0(p)^{-\epsilon} - f + T(p^{1-\epsilon} - cp^{-\epsilon} - f) \Rightarrow \frac{\mathrm{d}\Pi(p^0(p),p)}{\mathrm{d}p} = [(1-\epsilon)p^0(p)^{-\epsilon} + \epsilon cp^0(p)^{-1-\epsilon}]\frac{\mathrm{d}p^0(p)}{\mathrm{d}p} + [(1-\epsilon)p^{-\epsilon} + \epsilon cp^{-1-\epsilon}]T \Rightarrow \frac{\mathrm{d}\Pi(p^0(p),p)}{\mathrm{d}p}|_{p=p^s} = [(1-\epsilon)p^{s-\epsilon} + \epsilon cp^{s-1-\epsilon}] * [\frac{\mathrm{d}p^0(p)}{\mathrm{d}p} + T]$. Since $(1-\epsilon)p^{s-\epsilon} + \epsilon cp^{s-1-\epsilon} > 0$ is always true, $\frac{\mathrm{d}\Pi(p^0(p),p)}{\mathrm{d}p}|_{p=p^s} < 0 \Leftrightarrow T < -\frac{\mathrm{d}p^0(p)}{\mathrm{d}p}|_{p=p^s} = \frac{1}{\epsilon}(\frac{p^{s\,1-\epsilon} - f}{c})^{\frac{-1-\epsilon}{\epsilon}}(1-\epsilon)\frac{p^{s-\epsilon}}{c} = \frac{1-\epsilon}{\epsilon}\frac{p^s}{c}$. □