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The Supply-Side Effects of Household Heterogeneity

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The Supply-Side Effects of Household Heterogeneity*

Benjamin Schwanebeck† and Luzie Thiel‡

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Abstract

Household heterogeneity has been shown to be an important driver of aggregate demand. In this research, we demonstrate that it also impacts the supply side. We build a model in which heterogeneous households vary in their extent to which they supply production factors (labor and capital). Our model offers novel results about the consequences of inequality for the supply side, showing that (i) inequality distorts the factor allocation leading to higher marginal costs, and (ii) inequality becomes part of the Phillips curve. This is the "misallocation channel of inequality". The cyclicality of inequality crucially depends on how important capital is for production. Our findings have important implications for building models with household heterogeneity and for optimal monetary policy.

Keywords: Household Heterogeneity, Inequality, Supply-Side Effects, Optimal Monetary Policy, Factor Misallocation
JEL codes: E52, E61, E32, D24

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1 Introduction

Recent research documents that about 30% of households are financially-constrained "hand-to-mouth" households with near zero liquid wealth (Ampudia et al. 2018, Kaplan et al. 2014). An extensive and growing literature has studied the demand-side effects of inequality brought about by these financially-constrained households (e.g., Acharya et al. 2023, Auclert 2019, Bilbiie et al. 2022, Kaplan et al. 2018, Schwanebeck and Thiel 2024), but their consequences on the supply side are still vastly understudied. From a policy perspective, this gap in the literature is particularly pressing, as the economic effects of the COVID-19 pandemic and the ongoing energy crisis have brought supply-side factors back into sharp focus. The main goal of our project is to fill this gap.

In this paper, we provide novel theoretical evidence on the supply-side effects of inequality. The fundamental building block of our model is that in presence of between-households inequality, only a fraction of households provide capital to firms, while all household types provide labor input. We implement this heterogeneity in the provision of production factors into a tractable Heterogeneous Agent New Keynesian (HANK) model in the spirit of Bilbiie (2024) and uncover three main findings about the role of inequality for aggregate supply. First, inequality leads to factor misallocation, making the use of capital relatively more expensive to firms (misallocation channel of inequality). The supply-side effects of inequality, hence, depend on the capital intensity of production. Second, by raising firms' marginal cost, inequality also enters the Phillips curve as a cost driver. Third, we demonstrate that this misallocation channel of inequality also exerts countercyclical effects for reasonable calibrations, substantiating recent empirical evidence on the role of inequality in the business cycle (Bilbiie et al. 2023).

Our model features different household types, saver and hand-to-mouth households. Hand-to-mouth households are non-participating in financial markets, and thus only saver households provide capital to firms as a second production factor. We implement capital in a tractable way to allow for optimal policy analysis, following Carlstrom et al. (2010) and De Paoli and Paustian (2017). The heterogeneous supply of production factors matters for the design of optimal monetary policy. We derive a quadratic welfare function and find that the misallocation channel of inequality affects the central bank's trade-off between inequality, price and output stabilization such that the relative importance of each objective changes. In a tractable HANK model without capital as a second production factor, only some households receive profit income leading to inequality. This household heterogeneity represents a distortion implying welfare losses since a central bank targeting price stabilization now only reaches a fraction of households directly compared to a standard New Keynesian model with a representative household. Inflation stabilization becomes relatively less important. In our tractable HANK model with capital, profits also depend on inequality through the misallocation channel of inequality. Rising inequality decreases profits, and hence mitigates the welfare losses caused by household heterogeneity. Inflation stabilization becomes more relevant again. Overall, welfare losses caused by household inequality seem to be overstated in tractable HANK models with only labor as input.

We show that the cyclicality of inequality is decisive for the welfare analysis and crucially depends on the misallocation channel of inequality. This channel determines whether and to what extent inequality behaves procyclical (increasing in a boom) or countercyclical (decreasing in a boom) and as a consequence how inequality impacts inflation. While rising inequality is deflationary for countercyclical inequality, it is inflationary for procyclical inequality. We emphasize that the implementation of capital as a second production factor besides labor can lead to acyclical (independent of business cycle) or even procyclical inequality, whereby we argue that countercyclical inequality still remains the more relevant case. The more important the misallocation channel, the more the cyclicality of inequality turns from countercyclical to procyclical. As a consequence, the central bank can react more restrictively to a cost-push shock relative to the predictions of prior models that did not feature this channel. We conclude that the more active the misallocation channel is, the more effective monetary policy becomes in reducing welfare losses. We show that this finding is independent of whether the central bank conducts optimal policy under discretion or commitment. For the more relevant case of countercyclical inequality, inflation stabilization becomes more important compared to a tractable HANK model without capital as a second production factor.

Our results have a series of important implications, both for building models with household heterogeneity and for the design of optimal monetary policy. The misallocation channel of inequality impacts aggregate demand and supply dynamics, yet it is missing in models that consider labor as the sole input factor. Given that this channel affects the cyclicality of inequality, an important feature of models with household inequality, it should be considered in models that incorporate household heterogeneity. Although the supply-side effects of inequality imply welfare losses per se, this opens the room for redirecting optimal monetary policy. Overall, monetary policy should be more restrictive in face of a cost-push shock and mitigates welfare losses that arise from higher household heterogeneity. Monetary policy should take into account the supply-side effects of inequality since they impact the effectiveness of monetary policy in reducing welfare losses.

Contribution to the Literature. The main contribution of our work is to study the supply-side effects of *inequality*, which have received very little attention in the HANK literature thus far. Our study specifically contributes to three strands of the literature. First, we connect to a broad literature that studies how monetary policy affects the supply side (e.g., Baqaee et al. 2024, González et al. 2024, Michaelis and Palek 2016). González et al. (2024) construct a model in which firms are heterogeneous in terms of their marginal product of capital. They find a capital misallocation channel of monetary policy: Expansionary monetary policy reallocates capital to more productive firms increasing total factor productivity (TFP). Thus, monetary policy can have supply-side effects by impacting TFP. González et al. (2024) stress that central banks should take into account the supply-side effects of monetary policy. Baqaee et al. (2024) study a similar misallocation channel which arises if the resource allocation is inefficient in equilibrium: Monetary easing, as a positive demand shock, redistributes resources towards high-markup firms which undersupply in the initial state. Since high-markup firms change their prices less frequently than the low-markup firms, the reallocation of resources enhances the output response. Their analysis indicates that this channel is also quantitatively important. In both contributions, monetary policy has supply-side effects by impacting TFP. We contribute to this literature by showing that central banks should also take into account the supply-side effects of inequality. In addition, we perform an optimal policy analysis, in contrast to Baqaee et al. (2024).

Second, our paper specifically contributes to the creation of tractable HANK, i.e. THANK, models and the modelling of the supply side. Bilbiie et al. (2022) build a THANK model with capital, implemented as a state variable in a standard way, and investment. While investment has demand-side effects, capital, as a second production factor, implies supply-side effects. Their study shows that income and capital inequality (interpreted as a kind of wealth inequality) are complementary regarding their effects on the demand side. However, the authors refrain from analyzing the implied supply-side effects of inequality, as their focus remains on the demand side. Based on US data, Bilbiie et al. (2023) estimate a THANK model with capital to analyze the relation between inequality and business cycle in more detail. The income inequality is counter-cyclical. Both contributions, Bilbiie et al. (2023) and Bilbiie et al. (2022), refrain from a normative analysis. The cyclicality of inequality plays an important role in models with household heterogeneity (see, e.g., Bilbiie 2020, Bilbiie 2024, Bilbiie et al. 2024). For example, in Bilbiie (2024) the central bank's insurance motive becomes increasingly important as inequality becomes more countercyclical. For the simulation of these models, inequality is mostly assumed to be countercyclical as it is the more plausible case (Bilbiie et al. 2024). In contrast to our paper, these contributions focus on the demand-side effects of inequality. Challe et al. (2017) show that imperfect insurance leads to a precautionary savings motive which can impact the demand as well as the supply side. Precautionary savings have two opposing effects: In a recession, saver households save more for rainy days, which dampens aggregate demand. But at the same time, higher savings affect the supply side, as the interest rate decreases and the supply of capital increases. This results in higher investment stimulating aggregate demand. While they focus on the effects of precautionary savings, we analyze the supply-side effects when a factor of production is heterogeneously supplied resulting from inequality. In addition, our paper shows that accounting for these supply-side effects impacts the cyclicality of inequality, which is decisive for optimal monetary policy.

Finally, we also speak to the literature that studies the optimal design of monetary policy in face of heterogeneous households (e.g., Acharya et al. 2023, Bilbiie 2024 or Schwanebeck and Thiel 2024). In face of inequality, a consumption-insurance motive arises for the central bank. It is welfare-enhancing to reduce inequality fluctuations. Most of these contributions, including Acharya et al. (2023) and Bilbiie (2024), study heterogeneity in a one-country model. Schwanebeck and Thiel (2024), building a two-country model, underline the importance of household heterogeneity across countries within a monetary union. However, an analysis of the supply-side effects is missing in these contributions. Our paper adds to this strand of the literature by showing that the inequality objective of the central bank crucially depends on how important capital is for production.

Structure. The remainder of this paper is structured as follows. Section 2 presents the model framework. Section 3 analyzes the misallocation channel of inequality, whereby section 3.1 studies the supply-side effects of inequality and section 3.2 the link between inequality and the business cycle. Section 4 analyzes how optimal monetary policy looks like when inequality has supply-side effects. Section 5 concludes.

2 Model

We use an analytically THANK model based on Bilbiie (2024) and implement a second production factor which is only provided by a fraction of households. The model contains different household types (constrained and unconstrained) facing idiosyncratic risk, intermediate and final goods producers, a government, and a central bank.[1]

Firms demand two production factors from the household sector: labor and capital utilization services. We interpret the second production factor as capital utilization services following Carlstrom et al. (2010) and De Paoli and Paustian (2017) to implement "capital" in an analytically more simplified and thus more tractable way. In contrast to them, we assume heterogeneity according to its supply as not all but only a fraction of households provide capital utilization services. Given theses assumptions, we keep the model tractable to allow for optimal policy analysis.

2.1 Households

Households j can belong to two household states (with $j \in \{S, N\}$): being a saver (S) or non-participating (N) household. Both consume final goods C_t^j and provide labor L_t^j to firms. Additionally, S-households participate in financial markets and provide capital utilization services u_t^S to firms. Therefore, this type of household is (financially) unconstrained. Households of type N, instead, are constrained, they consume all their period income living hand-to-mouth and only supply labor to the firm sector. The motivation to explicitly model N-households comes from recent empirical facts (e.g., Ampudia et al. 2018), which show that there is a fraction of hand-to-mouth households with near zero liquid assets which is not able to smooth consumption. As these households are non-participating in financial markets, they do not provide capital utilization services to firms.

Households face an idiosyncratic risk of switching between the two states from period t to $t+1$. We model the switching process by a Markov chain with exogenous transition probabilities. With probability α, an S-household stays in the unconstrained state and switches to the constrained state with $(1-\alpha)$. An N-household stays constrained with ρ and switches to the unconstrained state with $(1-\rho)$. The share of constrained households evolves according to:

$$\lambda = \frac{1-\alpha}{2-\alpha-\rho} \tag{1}$$

[1] Appendix B provides an overview of the (log-linearized) model equations.

As we normalize total population to one, $(1-\lambda)$ denotes the share of S-households.

Both household types are part of a family with a family head maximizing the utility of the family weighted by household shares. The family head pools resources and redistributes them equally within a household state, but is not able to do so between states. This results in perfect insurance within a state. However, there is imperfect insurance between states as the family head cannot transfer resources between states. Thus, there can be consumption inequality, denoted by $q_t = C_t^S / C_t^N \gtreqless 1$.

S-households can self-insure against the idiosyncratic risk of becoming constrained by buying bonds. Bonds, as the only liquid asset, are transferable between the two states and pay out the gross nominal interest rate R_t from t to $t+1$. Let B_t^j be the real bond holdings per capita of household state j at the beginning of period t. The real bond holdings at the end of period t, but before switching, are $\tilde{B}_{t+1}^j$. At the end of the period, households switch and enter the next period with $\tilde{B}_{t+1}^j$. Thus, bond holdings evolve according to:

$$\begin{aligned}(1-\lambda)B_{t+1}^S &= \alpha(1-\lambda)\tilde{B}_{t+1}^S + (1-\rho)\lambda\tilde{B}_{t+1}^N \\ \lambda B_{t+1}^N &= (1-\alpha)(1-\lambda)\tilde{B}_{t+1}^S + \rho\lambda\tilde{B}_{t+1}^N\end{aligned} \tag{2}$$

We use (1) to rearrange (2):

$$\begin{aligned}B_{t+1}^S &= \alpha\tilde{B}_{t+1}^S + (1-\alpha)\tilde{B}_{t+1}^N \\ B_{t+1}^N &= (1-\rho)\tilde{B}_{t+1}^S + \rho\tilde{B}_{t+1}^N\end{aligned} \tag{3}$$

Furthermore, the model features stocks and capital as illiquid assets. Only S-households own firms and provide capital utilization services u_t^S to them for production. Following Carlstrom et al. (2010) and De Paoli and Paustian (2017), the capital stock is fixed, but there are variable utilization costs in form of utility cost.

The family head chooses $\{C_t^S, C_t^N, L_t^S, L_t^N, u_t^S, \tilde{B}_{t+1}^S, \tilde{B}_{t+1}^N\}$ to maximize life-time utility weighted by household shares:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\lambda)\left(\frac{(C_t^S)^{1-\sigma}}{1-\sigma} - \frac{(L_t^S)^{1+\varphi}}{1+\varphi} - \frac{(u_t^S)^{1+\varphi}}{1+\varphi}\right) + \lambda\left(\frac{(C_t^N)^{1-\sigma}}{1-\sigma} - \frac{(L_t^N)^{1+\varphi}}{1+\varphi}\right)\right] \tag{4}$$

with β as discount factor, σ as inverse of the intertemporal elasticity of substitution, and φ as inverse Frisch elasticity.

The family head optimizes (4) subject to (3), non-negative bond holdings and households' budget constraints:

$$C_t^S + \tilde{B}_{t+1}^S = w_t L_t^S + r_t u_t^S + \frac{1}{1-\lambda}\Pi_{Y,t} + \frac{R_{t-1}}{\pi_t} B_t^S - T_t^S \tag{5}$$

$$C_t^N + \tilde{B}_{t+1}^N = w_t L_t^N + \frac{R_{t-1}}{\pi_t} B_t^N + T_t^N \tag{6}$$

with w_t as real wage, r_t as real price for capital utilization services, $\Pi_{Y,t}$ as firm profits, and $\pi_t = P_t/P_{t-1}$ as gross inflation rate. The terms T_t^j are transfer/tax payments between households as a governmental redistribution scheme.

The FOCs for bonds, labor and capital utilization services read:

$$1 \geq \beta E_t \left[\left(\frac{C_t^S}{C_{t+1}^S} \right)^{\sigma} \frac{R_t}{\pi_{t+1}} (\alpha + (1-\alpha) q_{t+1}^{\sigma}) \right] \quad or \quad \tilde{B}_{t+1}^S = 0, \tag{7}$$

$$1 \geq \beta E_t \left[\left(\frac{C_t^N}{C_{t+1}^N} \right)^{\sigma} \frac{R_t}{\pi_{t+1}} (\rho + (1-\rho) q_{t+1}^{-\sigma}) \right] \quad or \quad \tilde{B}_{t+1}^N = 0, \tag{8}$$

$$w_t = (L_t^S)^{\varphi} (C_t^S)^{\sigma}. \tag{9}$$

$$w_t = (L_t^N)^{\varphi} (C_t^N)^{\sigma}. \tag{10}$$

$$r_t = (u_t^S)^{\varphi} (C_t^S)^{\sigma}. \tag{11}$$

The first two equations correspond to the bond-holding choices of savers and non-participating households as they take into account the probability of switching states next period. We will focus on calibrations that allow for end-of-period bond holdings of savers, i.e. $\tilde{B}_{t+1}^S \geq 0$, while $\tilde{B}_{t+1}^N = 0$.[2] Saver households take into account that they might end up in the constrained state next period. A precautionary savings motive arises that increases with expected inequality.

Equations (9)-(11) are standard factor supply equations. Note that the supply of capital utilization services is determined by its price and the marginal utility of consumption of S-households, as they are the only household type providing this production factor.

Aggregate consumption is given by $C_t = (1-\lambda)C_t^S + \lambda C_t^N$, while aggregate labor is given by $L_t = (1-\lambda)L_t^S + \lambda L_t^N$ and aggregate capital utilization services by $u_t = (1-\lambda)u_t^S$.

2.2 Firms

The firm sector consists of intermediate and final goods firms.

[2] In equilibrium, bonds will be in zero net supply.

Intermediate Goods Firms. There is a continuum of competitive firms producing intermediate goods x_t with labor and capital utilization services as input factors:

$$x_t = u_t^{\zeta} L_t^{1-\zeta} \tag{12}$$

with $\zeta > 0$ denoting production elasticity of u_t.[3]

The intermediate goods firms sell x_t at the real price mc_t to final goods firms. Maximizing their profit function $\Pi_{x,t} = mc_t x_t - w_t L_t - r_t u_t$ delivers the FOCs for factors demand:

$$(1-\zeta) mc_t x_t = w_t L_t \tag{13}$$

and

$$\zeta mc_t x_t = r_t u_t. \tag{14}$$

Final Goods Firms. There is a continuum of monopolistically competitive final goods firms. A firm z produces final goods $Y_t(z)$ with intermediate goods as input factor according to $Y_t(z) = x_t(z)$. The demand is given by $Y_t(z) = (P_t(z)/P_t)^{-\epsilon} Y_t$ with ϵ as substitution elasticity between goods. The final goods firms maximize their life-time profits facing Rotemberg (1982) price adjustments costs, with $\nu > 0$:

$$\max_{P_t(z)} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^S \left[(1+\omega) P_t(z) Y_t(z) - P_t mc_t x_t(z) - \frac{\nu}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 P_t Y_t \right] \tag{15}$$

where $\Lambda_{0,t}^S = \beta^t (C_0^S / C_t^S)^{\sigma} P_0 / P_t$ denotes the discount factor of S-households, who are the owners of the firms. A subsidy ω corrects for firms' market power in steady state. Maximization and assuming symmetry across firms, $P_t(z) = P_t$, deliver the Phillips curve:

$$(\pi_t - 1)\pi_t = \beta E_t \left[\left(\frac{C_t^S}{C_{t+1}^S} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1)\pi_{t+1} + \frac{\epsilon}{\nu} \left(mc_t - \frac{\epsilon - 1}{\epsilon} (1+\omega) \right) \right]. \tag{16}$$

In aggregate, total production is $Y_t = u_t^{\zeta} L_t^{1-\zeta}$.

2.3 Government

We assume an optimal subsidy implying marginal cost pricing in equilibrium, $\omega = (\epsilon - 1)^{-1}$. Hence, there are no profits in steady state. We introduce lump-sum taxation to finance the

[3] For $\zeta = 0$, the model would collapse to one without capital.

subsidy: $T_t^F = \omega Y_t$. Thus, net profits of final goods firms read:

$$\Pi_{Y,t} = \left[1 + \omega - mc_t - \frac{\nu}{2}(\pi_t)^2 - 1\right] Y_t - T_t^F = \left[1 - \frac{\nu}{2}(\pi_t - 1)^2\right] Y_t - w_t L_t - r_t u_t. \quad (17)$$

Income inequality between households occurs as S-households receive profit income (as firm owners) and capital income. The government can redistribute these income streams from S-households paying T_t^S to N-households receiving T_t^N:

$$\begin{aligned} T_t^S &= \frac{\lambda}{1-\lambda} ru + \frac{\tau}{1-\lambda}\left(\Pi_{Y,t} + r_t u_t - ru\right) \\ T_t^N &= ru + \frac{\tau}{\lambda}\left(\Pi_{Y,t} + r_t u_t - ru\right) \end{aligned} \quad (18)$$

while running a balanced budget, i.e. $(1-\lambda)T_t^S = \lambda T_t^N$.

We assume full redistribution in steady state. This is reflected by the first terms of (18), $\lambda(1-\lambda)^{-1} ru$ and ru, implying $C^S = C^N = C$ and therefore $L^S = L^N = L$.[4] The last terms, depending on τ, display additional redistribution per period. For $\tau = 0$, there is only steady-state redistribution in every period. For $\tau > 0$, additional redistribution takes place. For $\tau = \lambda$, full redistribution, $C_t^S = C_t^N = C_t$, holds for all t. For $0 \leq \tau < \lambda$, the redistribution scheme is skewed towards S-households, while the opposite holds for $\lambda < \tau \leq 1$.

2.4 Market Clearing

While bonds are in zero net supply, factor and goods market clearing result in the resource constraint:

$$(1-\lambda)C_t^S + \lambda C_t^N = \left[1 - \frac{\nu}{2}(\pi_t - 1)^2\right] Y_t. \quad (19)$$

3 Misallocation Channel of Inequality

3.1 Supply-Side Effects of Inequality

We log-linearize the model by taking a first-order approximation around the efficient zero-inflation steady state with full steady-state redistribution ($\pi = 0, q = 1$). Goods market clearing implies $\hat{C}_t = \hat{Y}_t$, while aggregate consumption reads $\hat{C}_t = (1-\lambda)\hat{C}_t^S + \lambda \hat{C}_t^N$.[5] Aggregate labor is given

[4] This assumption still allows for $C_t^S \neq C_t^N$ and $L_t^S \neq L_t^N$.

[5] A "$\wedge$" is used to denote the log deviation of a variable from its steady-state value.

by $\hat{L}_t = (1-\lambda)\hat{L}_t^S + \lambda \hat{L}_t^N$. Log-linearizing (16) results in the following standard Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\epsilon}{\nu}\hat{mc}_t + \epsilon_t^{mc}, \tag{20}$$

where ϵ_t^{mc} displays a cost-push shock following an AR (1) process with persistence parameter ρ_ϵ^{mc} and i.i.d. shock term υ_t: $\epsilon_t^{mc} = \rho_\epsilon^{mc}\epsilon_{t-1}^{mc} + \upsilon_t$. We use the cost-push shock as an example for more broader economic disturbances, such as the recent energy price shock.

Combining the factor demand functions from firms (13) and (14) with the factor supply functions from households (9)-(11) leads to the following equation that relates the ratio of the production factors to inequality:

$$\hat{L}_t - \hat{u}_t = \frac{\sigma\lambda}{1+\varphi}\hat{q}_t, \tag{21}$$

where $\hat{q}_t = \hat{C}_t^S - \hat{C}_t^N$. An increase in inequality will lead to a relative decline in the supply of u_t. With rising inequality, S-households increase consumption. Holding everything else constant, this lowers their marginal utility of consumption leading to a relative decline in the supply of u_t (wealth effect), which makes the factor u_t (relatively) more expensive for firms. Hence, a misallocation of the production factors occurs. We call this the *misallocation channel of inequality.*

Combining (9)-(14) leads to Proposition 1:

Proposition 1 *Marginal costs are characterized by:*

$$\hat{mc}_t = \underbrace{(\sigma+\varphi)\hat{Y}_t}_{Standard} + \underbrace{\sigma\lambda\zeta\hat{q}_t}_{Misallocation\ channel}. \tag{22}$$

An increase in inequality leads to a misallocation of production factors and therefore increases marginal costs. The second term illustrates the ***misallocation channel of inequality****.*

With rising inequality, capital utilization services become relatively more expensive than labor. As a result, higher inequality implies higher marginal costs and therefore has an inflationary impact, holding everything else constant. Inequality has supply-side effects via this misallocation channel. Our identified channel is increasing in λ and ζ, as well as in σ. The greater λ, the fewer households provide capital utilization services making its usage more expensive in relative terms. The more important capital becomes for production (the higher ζ), the more important this misallocation channel becomes. In models without household heterogeneity, $\lambda = 0$, or capital utilization services, $\zeta = 0$, this misallocation channel drops out.

The misallocation channel of inequality crucially depends on the cyclicality of q_t. Let us assume a boom, $\hat{Y}_t > 0$. In case of procyclical inequality, $\partial \hat{q}_t / \partial \hat{Y}_t > 0$, we have a stronger upward pressure on marginal costs and inflation. Rising inequality has an additional inflationary impact as it amplifies the inflationary impact of rising aggregate demand. In case of countercyclical inequality, $\partial \hat{q}_t / \partial \hat{Y}_t < 0$, marginal costs rise less and inequality dampens the overall inflationary effect. Hence, inequality has a deflationary impact. Similar arguments hold in a downturn, $\hat{Y}_t < 0$. To sum up, the cyclicality of inequality is important as it drives inflation dynamics. We discuss the cyclicality of inequality in more detail in the next section.

3.2 Inequality and the Business Cycle

In order to analyze the cyclicality of inequality, we combine the log-linearized budget constraints of both household types (5) and (6) with factors demand (13) and (14), firm profits $\Pi_{Y,t}/Y = -\hat{mc}_t$, and fiscal redistribution (18):

$$\hat{C}_t^N = \chi \hat{Y}_t \quad \text{and} \quad \hat{C}_t^S = \frac{1 - \lambda \chi}{1 - \lambda} \hat{Y}_t \tag{23}$$

with

$$\chi \equiv 1 + \varphi \left(1 - \frac{\tau}{\lambda}\right) \kappa \quad \text{and} \quad \kappa \equiv \frac{1 - \zeta \frac{1+\sigma+\varphi}{\sigma+\varphi}}{1 - \zeta \frac{\sigma}{\sigma+\varphi} \left(1 - \varphi \frac{\lambda}{1-\lambda} (1 - \zeta) \left(1 - \frac{\tau}{\lambda}\right)\right)}$$

Now, we can express inequality as a function of the business cycle:

$$\hat{q}_t = \hat{C}_t^S - \hat{C}_t^N = \frac{1 - \chi}{1 - \lambda} \hat{Y}_t. \tag{24}$$

The key parameter that defines the elasticities of households' consumption to aggregate income and therefore the cyclicality of inequality is χ. With full redistribution, i.e. $\tau = \lambda$, it follows that $\chi = 1$, thus inequality is absent.

Consider the case without capital, i.e. $\zeta = 0$, which implies $\kappa = 1$. Our model boils down to a THANK version as in Bilbiie (2024) with identical parameter χ and identical implications. The cyclicality of inequality primarily depends on fiscal redistribution. If $0 \leq \tau < \lambda$, then $\chi > 1$ and inequality is countercyclical, $\partial \hat{q}_t / \partial \hat{Y}_t < 0$. The logic of the New Keynesian cross leads to a disproportional increase in N-households' consumption for an increasing Y_t which reduces inequality. If $\lambda < \tau \leq 1$, then $\chi < 1$ and inequality is procyclical, $\partial \hat{q}_t / \partial \hat{Y}_t > 0$.

Now consider the case without labor, i.e. $\zeta = 1$, which implies $\kappa = -1/\varphi$ and $\chi = \tau / \lambda$.

The only source of income for N-households is fiscal redistribution. The cyclicality of inequality reverses. If $0 \leq \tau < \lambda$, then $\chi < 1$ and inequality is procyclical. If $\lambda < \tau \leq 1$, then $\chi > 1$ and inequality is countercyclical. An increase in Y_t makes S-households better off and as long as N-households do not receive a disproportional share of this additional income via fiscal redistribution, inequality is procyclical. Otherwise it is countercyclical.

Let us restrain our focus here and concentrate in the following sections on $0 \leq \tau < \lambda$, which we think is the most realistic case regarding fiscal redistribution. Note that there is still plentiful redistribution, as there is perfect insurance in steady state, i.e. $C^S = C^N$. Then, the case without capital, $\zeta = 0$, always leads to countercyclicality, while the case without labor, $\zeta = 1$, results in procyclicality.

Let us now consider the general case of $0 \leq \zeta \leq 1$ described in Proposition 2.

Proposition 2 *a) The parameter χ is decreasing in ζ. b) If $\zeta < (\sigma+\varphi)/(1+\sigma+\varphi)$, then $\chi > 1$ and inequality is* ***countercyclical*** $\left(\partial \hat{q}_t / \partial \hat{Y}_t < 0\right)$*. c) If $\zeta > (\sigma+\varphi)/(1+\sigma+\varphi)$, then $\chi < 1$ and inequality is* ***procyclical*** $\left(\partial \hat{q}_t / \partial \hat{Y}_t > 0\right)$*. d) If $\zeta = (\sigma+\varphi)/(1+\sigma+\varphi)$, then $\chi = 1$ and inequality is* ***acyclical*** $\left(\partial \hat{q}_t / \partial \hat{Y}_t = 0\right)$*.*

With regard to a), an increase in Y_t increases labor demand, N-households' wages and consumption. Since production for this additional demand involves two production factors, the additional shift in labor demand and the corresponding wage increase are lower compared to a model without a second factor, i.e. $\zeta = 0$. In other words, χ will be lower. While S-households suffer due to lower profits as marginal costs increase, the misallocation channel of inequality dampens this increase in marginal costs when inequality is countercyclical. They also receive additional income since they supply capital utilization services. This leads to a lower relative increase in N's consumption (relative to the S-households). The reduction of inequality will be smaller due to the introduction of capital, i.e. $\zeta > 0$.[6] The higher ζ, the lower the share of labor in production, the lower are the gains of N-households. This decreasing effect of ζ on χ can be shown by

$$
\begin{aligned}
\frac{\partial \chi}{\partial \zeta} =& \varphi \left(1 - \frac{\tau}{\lambda}\right) \frac{\partial \kappa}{\partial \zeta} = - \frac{\varphi \left(1 - \frac{\tau}{\lambda}\right) \frac{(1+\sigma+\varphi)}{(\sigma+\varphi)}}{\left[1 - \zeta \frac{\sigma}{\sigma+\varphi} \left(1 - \varphi \frac{\lambda}{1-\lambda} (1-\zeta) \left(1 - \frac{\tau}{\lambda}\right)\right)\right]^2} \left[\frac{1+\varphi}{1+\sigma+\varphi}\right. \\
&\left. + \frac{\sigma\varphi}{\sigma+\varphi} \frac{\lambda}{1-\lambda} \left(1 - \frac{\tau}{\lambda}\right) \left(\left(\zeta - \frac{\sigma+\varphi}{1+\sigma+\varphi}\right)^2 + \frac{\sigma+\varphi}{(1+\sigma+\varphi)^2}\right)\right],
\end{aligned}
\tag{25}
$$

[6] A similar argumentation holds in the procyclical case.

which is clearly negative for $\tau < \lambda$, the most relevant case.

When b) $\zeta < (\sigma + \varphi)/(1 + \sigma + \varphi)$, then (23) and (25) imply $0 < \kappa < 1$ and $\chi > 1$. An increase in Y_t increases N-households' wages and consumption disproportionately due to the New Keynesian cross logic, resulting in countercyclical inequality. c) The higher the importance of capital utilization services, i.e. when $\zeta > (\sigma + \varphi)/(1 + \sigma + \varphi)$, then $\kappa < 0$ and $\chi < 1$. The procyclicality of inequality stems from the fact, that the additional income of the S-households due to supplying capital utilization services now leads to a disproportionately increase in their consumption.[7] d) When $\zeta = (\sigma + \varphi)/(1 + \sigma + \varphi)$, then $\kappa = 0$ and $\chi = 1$ as the additional income due to the supply of capital utilization services and the decrease in profits due to higher marginal costs cancel each other out. Irrespectively of the redistribution scheme, both households equally profit from an increase in demand via higher wages. Hence, inequality is acyclical.

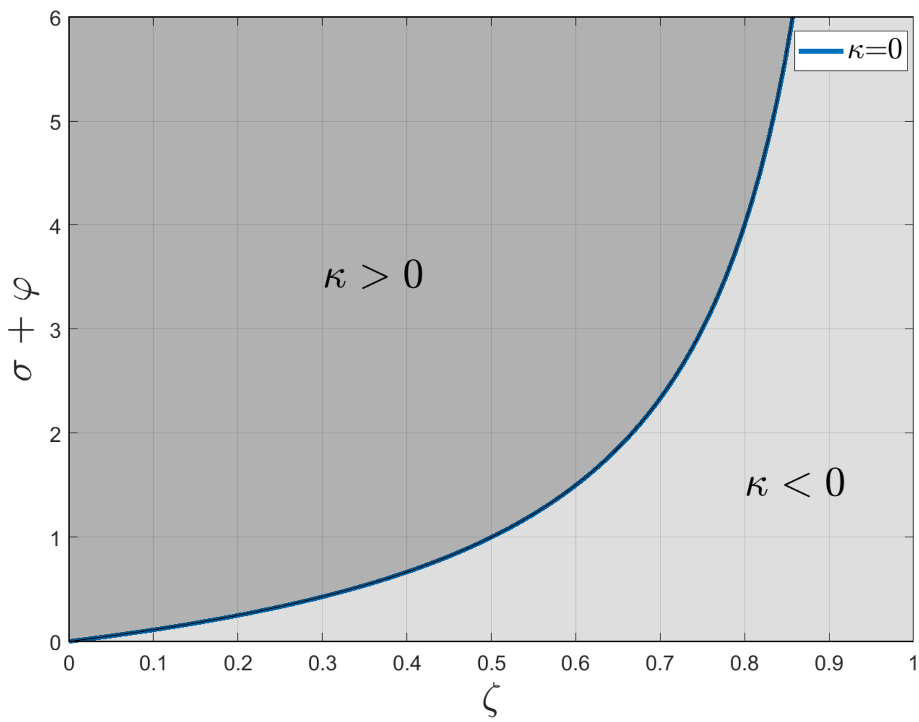

Figure 1: Dependence of κ on ζ and $\sigma + \varphi$.

How relevant are these cases? Other contributions assume countercyclical inequality for their model simulations by using $\tau < \lambda$, for example Bilbiie et al. (2024), as this seems to be the empirically more plausible case.[8] Whereas in our framework, the cyclicality depends on κ even if $\lambda < \tau$. Figure 1 displays how κ depends on ζ and the sum $\sigma + \varphi$. The standard values for the

[7] The opposite of b) and c) is true for $\lambda < \tau \leq 1$.

[8] See also Bilbiie et al. (2023) for an estimated THANK model. Based on US data, they show that income inequality is countercyclical.

sum lie in the range $1 \leq \sigma + \varphi \leq 3$. In order to have $\kappa < 0$, ζ must be larger than 0.5. This seems to be a rather large value for the production elasticity of capital utilization services. Nevertheless, one could also think of u_t as a composite of any additional production factors (e.g. capital, land, skill) that is solely supplied by one of the two household types. In this regard, $\zeta \geq 0.5$ can be reasonable, although it is unlikely. However, if $\sigma + \varphi < 1$, then ζ could be even lower than 0.5 to have $\kappa \leq 0$. Following Woodford (2003), the intertemporal elasticity of substitution should be relatively large, i.e. $\sigma < 1$, in models without a capital stock as state variable and investment spending, as it is the case for our model.[9] For example, Woodford (2003) sets $\sigma = 0.16$. In this regard, e.g., $\sigma + \varphi = 0.5$ would imply $\zeta \geq 1/3$ in order to have $\kappa \leq 0$. This would mean that the most commonly used value of 0.33 for the production elasticity of capital utilization services leads to acyclicality. To sum up, countercyclicality is still the more relevant case, but the incorporation of a second production factor can lead to acyclical or procyclical behavior of inequality.

3.3 Implications for the Transmission of Monetary Policy

Let us now obtain the aggregate Euler equation by combining the log-linearized version of (7) with (23):

$$\hat{Y}_t = \delta E_t \hat{Y}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)}(\hat{R}_t - E_t \hat{\pi}_{t+1}), \tag{26}$$

where $\delta \equiv 1 + (\chi - 1)\frac{1-\alpha}{1-\lambda\chi}$. Following Bilbiie (2024), we restrain our focus on $\lambda\chi < 1$. Hence, the interest rate elasticity, i.e. the term in front of $\hat{R}_t$, is negative. In addition, the question whether $\delta < 1$ or $\delta > 1$ exclusively depends on the term $(\chi - 1)$. As ζ affects χ, we can derive Proposition 3.

Proposition 3 *The misallocation channel of inequality makes monetary policy less effective in affecting aggregate demand as the effect of a one-time cut in the policy rate is decreasing in ζ.*

This follows from

$$\frac{\partial^2 \hat{Y}_t}{\partial(-\hat{R}_t)\partial\zeta} = \frac{\lambda(1-\lambda)}{\sigma(1-\lambda\chi)^2}\frac{\partial\chi}{\partial\zeta} < 0, \tag{27}$$

which is clearly negative, given Proposition 2. An interest rate cute initially increases aggregate demand through S-households' demand, thereby raising labor demand, which also increases N-households' wages and consumption. This boosts aggregate demand further and in the case of countercyclicality, i.e. $\chi > 1$, amplifies the overall effect of an one-time cut in the interest rate

[9] Carlstrom et al. (2010) also point this out.

compared to a model with savers only.[10] However, as described above, the shift in labor demand is lower due to the second factor while S-households suffer less due to the smaller increase in marginal costs and the additional capital income. Hence, compared to a THANK model without a second production factor, the effect of monetary policy on aggregate demand decreases.

By analyzing the effect of an interest rate cut on future aggregate demand, we tip into the territory of the forward guidance puzzle (Del Negro et al. 2023). Similar to, for instance, Bilbiie (2020), Bilbiie (2024), or Bilbiie et al. (2022), we can derive Proposition 4. In contrast to their contributions, χ also depends on κ in our model framework as already pointed out in Proposition 2.

Proposition 4 *The aggregate Euler equation is characterized by a)* ***compounding****, i.e.* $\delta > 1$*, if* $\chi > 1$*, which aggravates the forward guidance puzzle, or b)* ***discounting****, i.e.* $\delta < 1$*, if* $\chi < 1$*, which resolves the forward guidance puzzle.*

The proof for the forward guidance puzzle can be easily found by forward-solving (26). A decrease in the interest rate at $t + T$ has the effect

$$\frac{\partial \hat{Y}_t}{\partial(-\hat{R}_{t+T})} = \frac{1-\lambda}{\sigma(1-\lambda\chi)}\delta^T, \tag{28}$$

which is clearly increasing in T, if $\delta > 1$, or decreasing in T, if $\delta < 1$.

By shutting our misallocation channel off, i.e. $\zeta = 0$ resulting in $\kappa = 1$, Proposition 2 implies countercyclicality, i.e. $\chi > 1$. It immediately follows that $\delta > 1$. Compounding occurs since good news about future income, i.e. $E_t Y_{t+1}$, means more consumption today and lower expected inequality. The latter one lowers the precautionary savings motive. Combined with zero net holdings of bonds, households increase consumption even more and the New Keynesian cross logic applies.

Misallocation Channel of Inequality. Let us now turn the misallocation channel of inequality on, i.e. $0 < \zeta \leq 1$. Proposition 2 shows that income inequality can become procyclical, i.e. $\chi < 1$, even if $\tau < \lambda$, which is absent in the contributions cited above. In this case, $\delta < 1$. Good news about future income means more consumption today and higher expected inequality. This increases the precautionary savings motive, resulting in a lower than one-to-one increase in overall consumption, i.e. discounting. The acyclical case of Proposition 2, i.e. $\chi = \delta = 1$, leads

[10] In a Representative Agent New Keynesian (RANK) model, i.e. $\lambda = 0$, $\partial\hat{Y}_t/\partial(-\hat{R}_t) = 1/\sigma$. Under countercyclicality, this term is larger in our THANK model compared to a RANK model.

to the standard textbook version of the Euler equation. Hence, in the cases of procyclicality or acyclicality, the misallocation channel of inequality resolves the forward guidance puzzle.

Our findings complement the HANK literature which has so far focused primarily on how inequality affects aggregate demand (as McKay and Wolf 2023 summarizes). In our model framework, inequality also has an effect on aggregate supply since one of the two factors is solely supplied by one of the two household types. Fluctuations in inequality lead to an inefficient allocation between the two factors. This supply-side effect is missing in HANK models without a second production factor like capital utilization services. The cyclicality of inequality is also affected by the second production factor. The misallocation channel of inequality has implications for monetary policy and thus for the design of optimal policy that we discuss in more detail in the next section.

4 Optimal Monetary Policy

In this section, we analyze the implications of the misallocation channel of inequality for optimal monetary policy. First, we derive a welfare function in section 4.1 and show that an additional welfare target for the central bank arises in face of capital, $\zeta > 0$. In section 4.2, we look at optimal monetary policy under discretion and commitment. In section 4.3, we show how the central bank should optimally react to economic shocks in face of supply-side effects of inequality.

4.1 Welfare Function

The central bank conducts Ramsey optimal policy by setting the nominal interest rate in order to maximize the weighted aggregate of households' period utility functions:

$$\psi_t = (1-\lambda)U(C_t^S, u_t^S, L_t) + \lambda U(C_t^N, L_t) \tag{29}$$

We take a second-order approximation around the efficient steady state and drop terms of third or higher order.[11]

Proposition 5 *The central bank maximizes the following welfare objective:*

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Psi_t + t.i.p. \tag{30}$$

[11] Appendix A describes the derivation in detail.

with t.i.p. as terms independent of policy and where the per-period loss is given by

$$\Psi_t = (\sigma+\varphi)\hat{Y}_t^2 + \nu\hat{\pi}_t^2 + \lambda(1-\lambda)(\sigma+\varphi)\frac{\sigma}{\varphi}\Omega\hat{q}_t^2,$$

where $\Omega \equiv 1 - \underbrace{\zeta\frac{\sigma}{\sigma+\varphi}\left(1-\frac{\lambda}{1-\lambda}\frac{\varphi}{1+\varphi}(1-\zeta)\right)}_{new}.$

The first two terms of Ψ_t, output and price stabilization, are standard targets. The third term arises in face of household heterogeneity ($0 < \lambda < 1$) and leads to an inequality objective for the central bank. For $\zeta = 0$ resulting in $\Omega = 1$, we get the standard targets for a THANK model as in Bilbiie (2024). The inequality objective ultimately puts a smaller relative weight on inflation. The reasoning for this is the following: As only S-households receive profit income, the importance of stabilizing profits via inflation stabilization declines.

The *new* welfare weight $\Omega \neq 1$ additionally appears for $\zeta > 0$. The first part leads to a lower weight on inequality, while the last part increases the weight. This last part reflects the welfare loss per se, since any fluctuation of q_t distorts the factor allocation between labor and capital utilization services. On the other hand, there is an opposite effect. The misallocation channel shows that profits are directly influenced by inequality. This makes profit stabilization via inflation stabilization more important. Hence, the weight on inequality decreases, i.e. the relative weight on inflation increases. This is reflected by the first part of the *new* welfare weight on inequality.

The effect of ζ on the new welfare weight Ω is given by

$$\frac{\partial\Omega}{\partial\zeta} = -\frac{\sigma}{\sigma+\varphi}\left[1-\frac{\lambda}{1-\lambda}\frac{\varphi}{1+\varphi}(1-2\zeta)\right]. \tag{31}$$

which is negative for $\zeta > 0$ if $\lambda < (\varphi+1)/(2\varphi+1)$, which holds true for any reasonable calibration.[12] Thus, inequality stabilization becomes relatively less important for the central bank due to the second production factor.

In order to take into account how inequality depends on the business cycle, we can use (24) to rewrite the per-period loss as

$$\Psi_t = (\sigma+\varphi)\varpi\hat{Y}_t^2 + \nu\hat{\pi}_t^2, \tag{32}$$

[12] Standard values for φ lie in the range of $0 \leq \varphi \leq 2$. Hence, λ must be at least lower than 0.6, which is fairly reasonable. Given this, $\Omega > 0$.

where $\varpi = 1 + \frac{\lambda}{1-\lambda}(\chi - 1)^2 \frac{\sigma}{\varphi}\Omega$.

As long as inequality is cyclical, i.e. $\chi \neq 1$, the inequality objective increases the importance of output stabilization as $\varpi > 1$. The question arises, whether the misallocation channel of inequality additionally increases the importance of output stabilization compared to $\zeta = 0$ and thus $\Omega = 1$. This ultimately depends on the cyclicality of inequality. The effect of ζ on ϖ is given by

$$\frac{\partial \varpi}{\partial \zeta} = \frac{\lambda}{1-\lambda}\frac{\sigma}{\varphi}\left(2(\chi - 1)\Omega \frac{\partial \chi}{\partial \zeta} + (\chi - 1)^2 \frac{\partial \Omega}{\partial \zeta}\right), \tag{33}$$

Since χ and Ω are decreasing in ζ, the overall effect is negative, i.e. $\partial \varpi / \partial \zeta < 0$, if inequality is countercyclical, i.e. $\chi > 1$, which is the more relevant case. Thus, the importance of output stabilization compared to $\zeta = 0$ and $\Omega = 1$ decreases due to the misallocation channel of inequality. This is a result of the inflation-dampening impact of countercyclical inequality described in section 3.1.

However, if inequality is procyclical, i.e. $\chi < 1$, the importance of output stabilization compared to $\zeta = 0$ and $\Omega = 1$ may increase. The first term in parentheses in (33) is now positive, while the second one is still negative. This results from the upward pressure on marginal costs and the corresponding inflationary impact of procyclical inequality described in section 3.1. As both effects work in opposite directions, the importance of output stabilization, ϖ, could increase, decrease or remain unchanged compared to the case without the misallocation channel. In sum, the implications of this channel for the procyclical case are not clear-cut.

4.2 Discretion versus Commitment

We derive the optimal policy responses of the central bank under discretion and commitment when production factors are heterogeneously supplied by households.

By using (22) and (24), we can rewrite the Phillips curve (20) as

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\epsilon}{\nu}(\sigma + \varphi)\mu \hat{Y}_t + \epsilon_t^{mc}, \tag{34}$$

where $\mu = 1 + \frac{\sigma}{\sigma+\varphi}\frac{\lambda}{1-\lambda}(1 - \chi)\zeta$.

Assuming that the central bank is not able to commit to any future policy plan, the optimization problem is to sequentially solve (30) subject to (34), taking expectations as given.

Proposition 6 *The condition for optimal monetary policy under discretion is given by*

$$\hat{Y}_t = -\epsilon \frac{\mu}{\varpi} \hat{\pi}_t, \tag{35}$$

while inflation and output in equilibrium read

$$\hat{\pi}_t = \frac{\nu\varpi}{(\sigma+\varphi)(\epsilon\mu)^2 + \nu\varpi(1-\beta\rho_\epsilon^{mc})}\epsilon_t^{mc} \quad \text{and} \quad \hat{Y}_t = -\frac{\epsilon\nu\mu}{(\sigma+\varphi)(\epsilon\mu)^2 + \nu\varpi(1-\beta\rho_\epsilon^{mc})}\epsilon_t^{mc}. \tag{36}$$

The following interest rate rule implements this equilibrium:

$$\hat{R}_t = \left[1 + \left(\frac{1}{\rho_\epsilon^{mc}} - \delta\right)\frac{\epsilon\mu}{\varpi}\frac{\sigma(1-\lambda\chi)}{1-\lambda}\right] E_t\hat{\pi}_{t+1}. \tag{37}$$

Inserting (35) into (34) and iterating forward leads to (36), while using the latter in (26) results in (37). Proposition 6 nests the standard textbook case for $\lambda = 0$, in which $\mu = \varpi = 1$, and the case for a THANK model as in Bilbiie (2024) for $\zeta = 0$, in which $\mu = 1$ and $\varpi > 1$. Comparing the latter with the former case reveals that it is optimal that output responds less to a given increase in inflation. In other words, due to inequality, it is optimal to have lower output volatility and more fluctuations in inflation relative to the standard textbook case of $\lambda = 0$. In case of countercyclical inequality and according to Proposition 4 a) compounding, i.e. $\delta > 1$, the interest rate increase due to a cost-push shock is more moderate. It is even possible to have an interest rate cut, if there is enough compounding. In case of procyclicality and thus $\delta < 1$, the interest rate increase is even stronger than in the standard textbook case.

If $\zeta > 0$, the overall effect depends on the cyclicality of inequality. As discussed in section 4.1, the effect of ζ on ϖ is negative for $\chi > 1$, but ambiguous for $\chi < 1$. However, $\mu < 1$ if inequality is countercyclical and $\mu > 1$ if inequality is procyclical. Hence, in both cases, there are no clear-cut analytical results, but we can infer the following remarks. In both cases, monetary policy is less effective in changing aggegrate demand, see Proposition 3. If inequality is countercyclical, it is also less effective in affecting inflation ($\mu < 1$), but output volatility leads to lower losses ($\partial\varpi/\partial\zeta < 0$). Hence, compared to a THANK model without supply-side effects of inequality, the central bank must use the interest rate more aggressively - and can do so as the welfare weight on output disturbances is lower - for the same change in inflation. In other words, the model economy moves towards the standard textbook case and the misallocation channel can mitigate the distortions from inequality. In the case of procyclical inequality, the central bank is less effective in changing output, but output changes have a larger impact on inflation ($\mu > 1$). However, the welfare effects are ambiguous.

Assuming that the central bank is fully able to commit to any future policy plan, the optimization problem is to solve (30) subject to (34).

Proposition 7 *The condition for optimal monetary policy under commitment is given by*

$$\hat{Y}_t - \hat{Y}_{t-1} = -\epsilon \frac{\mu}{\varpi} \hat{\pi}_t. \tag{38}$$

As in the standard textbook case without heterogeneity, optimal monetary policy under commitment targets the price level. Similar to discretion, there are no clear-cut analytical results for the effects of ζ. Hence, we rely on simulations.

Simulations. For simulating the model, we use the calibration summarized by Table 1. Most of the parameter values are commonly used in the THANK literature (see, e.g., Bilbiie and Ragot 2021). For the sake of simplicity, we focus (for the moment) on a transitory cost-push shock of $\upsilon_1 = 0.01$.[13] The time interval is a quarter.

Table 1: Calibration

Households		
β	0.98	Discount factor
σ	1	Inverse intertemporal subsitution elasticity
φ	0.25	Inverse Frisch elasticity
λ	$[0, 1]$	Share of constrained households
α	0.9	Idiosyncratic risk
ρ	$2 - \alpha - (1 - \alpha)/\lambda$	Idiosyncratic risk
τ	0	Redistribution
Firms		
ϵ	6	Substitution elasticity between goods
ζ	$[0, 1]$	Importance of capital for production
ν	100	Rotemberg price adjustment cost
Cost-push shock		
$\rho_{\epsilon^{mc}}$	$(0; 0.8)$	Persistence

Figure 2 shows the welfare losses depending on λ and ζ according to (30) under discretion (left panel) and commitment (right panel). Losses are expressed in percentage of steady-state consumption as a result of deviations from steady state due to the cost-push shock.[14]

[13] The interest rate rule (37) then becomes $\hat{R}_t = \frac{\epsilon\mu}{\varpi} \frac{\sigma(1-\lambda\chi)}{1-\lambda} \hat{\pi}_t$ as $\rho_\epsilon^{mc} = 0$.

[14] Similar patterns emerge for different values of σ and ρ_ϵ^{mc}. Results are available upon request.

Both panels show the standard textbook case at $\lambda = \zeta = 0$. An increase in λ, holding $\zeta = 0$, leads to higher welfare losses. However, when we introduce ζ, welfare losses decrease under discretion as well as under commitment. This stems from the fact that the central bank allows for more output fluctuations in favor of stabilizing more inflation.

As described above, when inequality is countercyclical,[15] the welfare losses due to output volatility decrease, i.e. ϖ decreases. On the other hand, the impact of output on inflation shrinks, i.e. $\mu < 1$. Taking both effects together, monetary policy uses the interest rate more actively. The simulations show that monetary policy becomes more effective in reducing welfare losses compared to THANK models without capital.

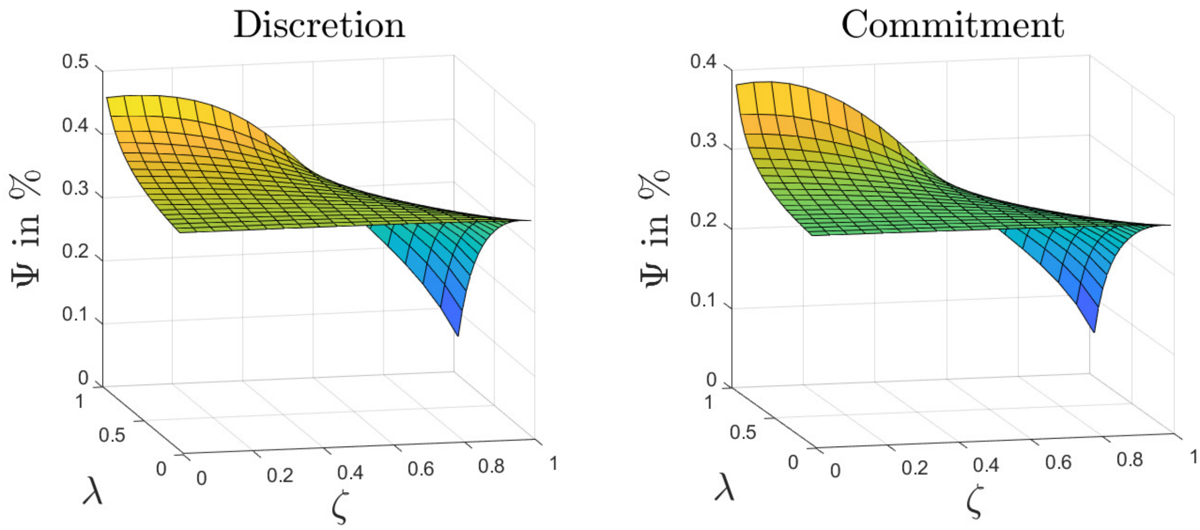

Figure 2: Welfare losses Ψ for a cost-push shock for discretion and commitment policy dependent on λ and ζ (for $\tau = 0$ and $\rho_{\epsilon}^{mc} = 0$).

Let us turn to the less relevant case of procyclical inequality.[16] In order to fight inflation, the necessary decline in output is lower, since the corresponding decline in inequality is deflationary. In this scenario, losses are decreasing in λ and ζ as the misallocation channel of inequality becomes more prominent. There are even welfare gains compared to the standard textbook case due to the higher effectiveness of monetary policy in stabilizing inflation.

By including a second production factor, inequality can lead to an inefficient factor allocation. Although this distorted allocation implies welfare losses per se, it opens the room for redirecting optimal monetary policy as inequality impacts the supply side directly. Overall, monetary policy is more effective in mitigating welfare losses due to inequality. This leads to profound insights regarding the analysis of THANK models: By including only one production factor, welfare

[15] Given our calibration, this is the case for $\zeta < 5/9$.
[16] Given our calibration, $\zeta > 5/9$ leads to procyclicality.

losses due to inequality seem to be overstated.

4.3 Optimal Response to Economic Shocks

In this section, we want to show in detail how the central bank optimally reacts to a cost-push shock in face of supply-side effects of household heterogeneity, i.e. the misallocation channel of inequality. As the capital intensity of production turns out to be decisive for the cyclicality of inequality, we simulate the model for different values of ζ. We set the share of constrained households to $\lambda = 0.3$ matching the estimated average value for the euro area (Almgren et al. 2022). In addition, the persistence parameter ρ_{ϵ}^{mc} is set to 0.8.

Figure 3 shows the impulse response functions (IRFs) after a cost-push shock of $\upsilon_1 = 0.01$ under Ramsey optimal monetary policy, i.e. commitment. The IRFs depict log-deviations from steady state.

Figure 3 emphasizes our analytical results from section 3.2. The cyclicality of inequality crucially depends on ζ, see panel A and B. In a THANK model without capital ($\zeta = 0$, black solid line), inequality is countercyclical. This is also the case for $\zeta = 0.25$ (black dashed line) and $\zeta = 0.5$ (blue line), although the deflections of the IRFs are much smaller. The greater ζ, the less countercyclical inequality reacts. It becomes procyclical for a more capital-intensive production ($\zeta = 0.75$, red dashed line, and $\zeta = 1$, red solid line), with the largest deflection for a THANK model without labor ($\zeta = 1$). This directly follows from Proposition 2. Given our calibration, the acyclical case is at $\zeta = 5/9$. Throughout the increase in ζ, welfare losses are declining.

As described in the previous section, the capital-intensity is also decisive for optimal monetary policy. In all five scenarios, the central bank optimally reacts expansionary in the first period following a cost-push shock. However, the optimal response is the more restrictive, the more capital-intensive production is, see panel C. This is related to the cyclicality of inequality. When inequality becomes procyclical ($\zeta = 0.75$ or $\zeta = 1$), the central bank reacts more restrictively and thus can weaken inflationary dynamics more effectively, see panel D. Hence, inflation fluctuations are lowest for $\zeta = 1$. Inflation rises most strongly in the case of $\zeta = 0$. It is about 24.47% higher compared to $\zeta = 1$ in the first period.

This simulation analysis emphasizes the importance of ζ for optimal monetary policy. As the cyclicality determines the impact of inequality on inflation, it matters for the design of optimal monetary policy. By including the misallocation channel of inequality, the central bank focuses more on inflation stabilization compared to a single-production-factor THANK model.

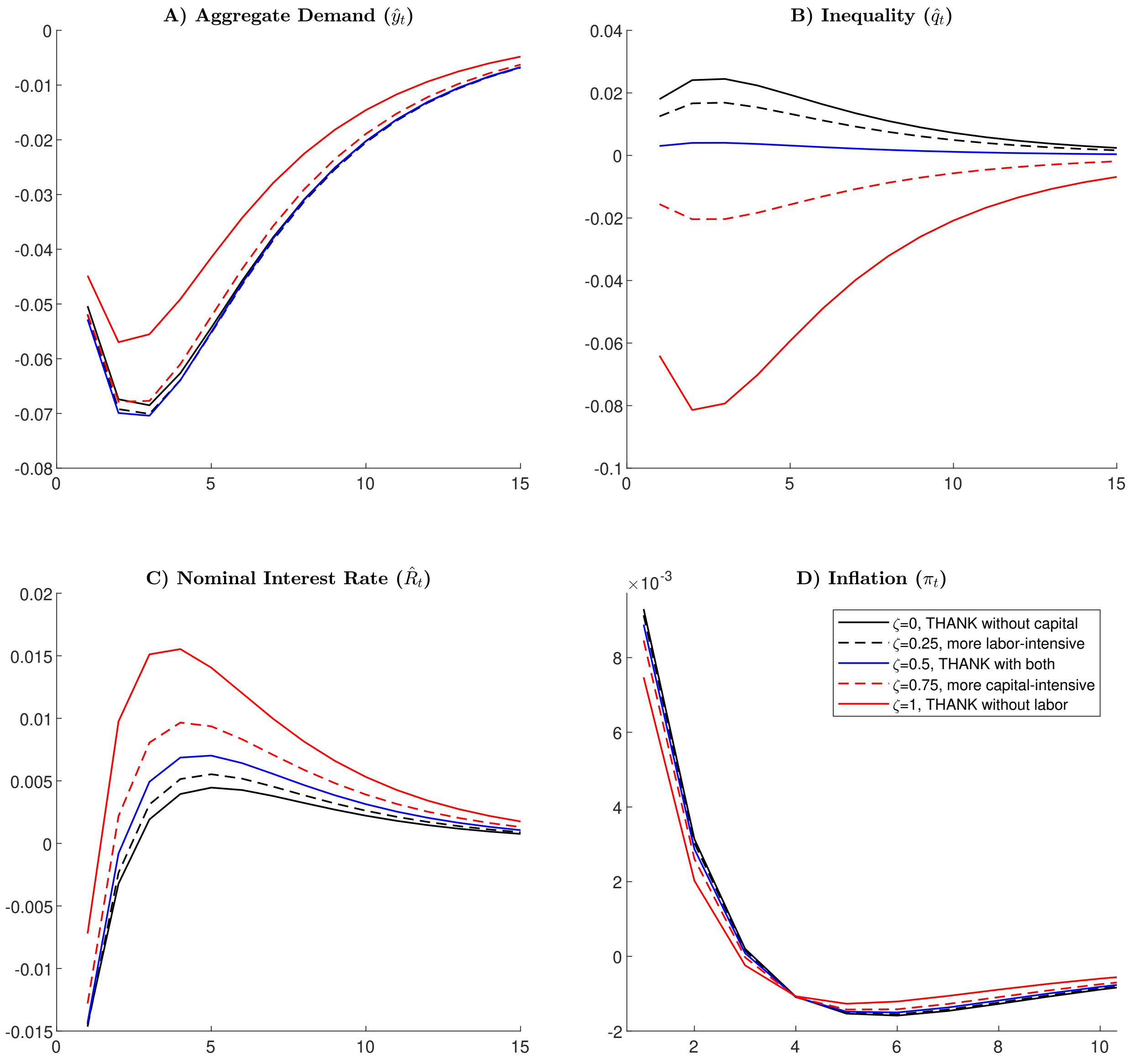

Figure 3: Impulse response functions (IRFs) to a cost-push shock for different ζ values under Ramsey optimal monetary policy. The IRFs depict log-deviations from steady state.

5 Conclusion

In this paper, we show that household inequality drives the supply side, alongside the well-known and intensively researched demand-side effects. Inequality impacts firms' marginal costs if only a fraction of households provides certain production factors, such as capital. To analyze the supply-side effects of inequality, we extend a THANK model based on Bilbiie (2024) by capital utilization services as a second production factor to enrich the supply side. We demonstrate that the heterogeneous supply of capital has important consequences for the firm side as well as optimal monetary policy.

Our main results are: First, inequality becomes part of marginal costs and hence affects inflation dynamics. We frame this as the misallocation channel of inequality as an increase in household heterogeneity distorts the factor allocation between labor and capital utilization services. Second, the more capital-intensive the production is, the less countercyclical inequality becomes. For very capital-intensive production, inequality can even become procyclical. Third, although the misallocation channel itself implies welfare losses per se, it opens the room for redirecting optimal monetary policy and mitigates welfare losses that arise from higher household heterogeneity. Overall, monetary policy should be more restrictive in face of a cost-push shock.

Future research could extend the framework in various ways: One could implement a banking sector with financial frictions as in De Paoli and Paustian (2017) to analyze how financial stability and inequality interact, a field that is still understudied (Colciago et al. 2019). Furthermore, one could enrich the supply side by heterogeneous firms (Moll 2014) or extend the framework to a currency union (Palek and Schwanebeck 2019, Schwanebeck and Thiel 2024). Another extension could be to endogenize the risk of switching the household type as in Ravn and Sterk (2021) or Thiel (2024) to allow for time-varying idiosyncratic risk. Finally, our paper focuses on the extensive margin of providing capital to firms. Future research could analyze the intensive margin (e.g., higher saving rates through higher inequality).

Our analysis has important implications for building models with household heterogeneity and for monetary policy. We argue that a second production factor, capital utilization services, is a useful extension for models with heterogeneous households. It turns out to be decisive for the cyclicality of inequality, affecting the demand side, and for the factor allocation, affecting the supply side. In models in which labor is the only input factor, the misallocation channel of inequality is missing. The importance of the second production factor determines how strongly inequality reacts to shocks. Additionally, the design of optimal monetary policy changes if there is heterogeneous supply of production factors. In this case, monetary policy becomes more effective in mitigating welfare losses caused by inequality. By including only one production factor in THANK models, welfare losses due to inequality seem to be overstated.

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Appendix A: Derivation of the Welfare Function

Let X_t be a generic variable and X its steady state. Then, we define $\hat{X}_t$ as the log deviation of X_t around X, $\hat{X}_t \equiv \log(X_t/X)$. Hence, using a second-order approximation yields

$$\frac{X_t - X}{X} = \exp(\hat{X}_t) - 1 \simeq \hat{X}_t + \frac{1}{2}\hat{X}_t^2. \tag{A.1}$$

Let $U(C_t^S, u_t^S, L_t^S)$ and $U(C_t^N, L_t^N)$ be the period utility function of S and N households. Then the central bank's period loss function is given by a weighted sum of these utility functions:

$$\psi_t = (1-\lambda)U(C_t^S, u_t^S, L_t^S) + \lambda U(C_t^N, L_t^N). \tag{A.2}$$

We take a second-order approximation around the efficient zero-inflation steady state[1] and drop terms of third or higher order:

$$\begin{aligned}\frac{\psi_t - \psi}{C^{1-\sigma}} =&(1-\lambda)\left(\hat{C}_t^S + \frac{1-\sigma}{2}(\hat{C}_t^S)^2\right) + \lambda\left(\hat{C}_t^N + \frac{1-\sigma}{2}(\hat{C}_t^N)^2\right)\\ &-(1-\lambda)(1-\zeta)\left(\hat{L}_t^S + \frac{1+\varphi}{2}(\hat{L}_t^S)^2\right) - \lambda(1-\zeta)\left(\hat{L}_t^N + \frac{1+\varphi}{2}(\hat{L}_t^N)^2\right)\\ &-\zeta\left(\hat{u}_t + \frac{1+\varphi}{2}\hat{u}_t^2\right)\end{aligned} \tag{A.3}$$

We evaluate the resource constraint given by (19) at the efficient steady state. Therefore, we make use of $Y_t = (1-\lambda)u_t^{\zeta}L_t^{1-\zeta}$,$L_t = (1-\lambda)L_t^S + \lambda L_t^N$, $C = Y$ and $\pi = 1$ to get the following second-order approximation after rearranging:

$$\begin{aligned}(1-\lambda)\hat{C}_t^S + \frac{1-\lambda}{2}(\hat{C}_t^S)^2 + \lambda\hat{C}_t^N + \frac{\lambda}{2}(\hat{C}_t^N)^2 =& -\frac{\nu}{2}\hat{\pi}_t^2 + \zeta\left(\hat{u}_t + \frac{1}{2}\hat{u}_t^2\right) - \frac{1}{2}\zeta(1-\zeta)\hat{u}_t^2\\ &+(1-\lambda)(1-\zeta)\left(\hat{L}_t^S + \frac{1}{2}(\hat{L}_t^S)^2\right) + \lambda(1-\zeta)\left(\hat{L}_t^N + \frac{1}{2}(\hat{L}_t^N)^2\right)\\ &-\frac{1}{2}\zeta(1-\zeta)\left((1-\lambda)\hat{L}_t^S + \lambda\hat{L}_t^N\right)^2 + \zeta(1-\zeta)\hat{u}_t\hat{L}_t\end{aligned} \tag{A.4}$$

We insert (A.4) into (A.3):

$$\begin{aligned}\frac{\psi_t - \psi}{C^{1-\sigma}} =& -\frac{\sigma}{2}\left((1-\lambda)(\hat{C}_t^S)^2 + \lambda(\hat{C}_t^N)^2\right) - \frac{\varphi}{2}\left(\zeta\hat{u}_t^2 + (1-\zeta)\left((1-\lambda)(\hat{L}_t^S)^2 + \lambda(\hat{L}_t^N)^2\right)\right)\\ &-\frac{\nu}{2}\hat{\pi}_t^2 - \frac{1}{2}\zeta(1-\zeta)\left(\hat{L}_t^2 - 2\hat{L}_t\hat{u}_t + \hat{u}_t^2\right)\end{aligned} \tag{A.5}$$

[1] In the efficient steady state, it applies: $C^S = C^N = C$, $L^S = L^N = L$, $(1-\zeta) = L^{1+\varphi}(C^{1-\sigma})^{-1}$ and $\zeta(1-\lambda)^{-1} = (u^S)^{1+\varphi}(C^{1-\sigma})^{-1}$.

Up to first order, the FOCs regarding labor supply, i.e. equations (9) and (10), together with the definition of aggregate labor lead to $\hat{L}_t^S = \hat{L}_t - \lambda\frac{\sigma}{\varphi}\hat{q}_t$ and $\hat{L}_t^N = \hat{L}_t + (1-\lambda)\frac{\sigma}{\varphi}\hat{q}_t$. We make use of these equations and of $\hat{C}_t^S = \hat{C}_t + \lambda\hat{q}_t$, $\hat{C}_t^N = \hat{C}_t - (1-\lambda)\hat{q}_t$, and $\hat{C}_t = \hat{Y}_t$, in order to eliminate household-specific variables:

$$\begin{aligned}\frac{\psi_t-\psi}{C^{1-\sigma}} =& -\frac{\sigma}{2}\left(\hat{Y}_t^2 + \lambda(1-\lambda)\hat{q}_t^2\right) - \frac{\varphi}{2}\left(\zeta\hat{u}_t^2 + (1-\zeta)\left(\hat{L}_t^2 + \lambda(1-\lambda)\frac{\sigma^2}{\varphi^2}\hat{q}_t^2\right)\right)\\ &-\frac{\nu}{2}\hat{\pi}_t^2 - \frac{1}{2}\zeta(1-\zeta)\left(\hat{L}_t - \hat{u}_t\right)^2\end{aligned} \tag{A.6}$$

The term $\left(\zeta\hat{u}_t^2 + (1-\zeta)\hat{L}_t^2\right)$ can be replaced and rearranged by using a first-order approximation of the production function Y_t, given by $\hat{Y}_t = \zeta\hat{u}_t + (1-\zeta)\hat{L}_t$:

$$\zeta\hat{u}_t^2 + (1-\zeta)\hat{L}_t^2 = \hat{Y}_t^2 + \zeta(1-\zeta)\left(\hat{L}_t - \hat{u}_t\right)^2 \tag{A.7}$$

Using this and collecting terms lead to:

$$\begin{aligned}\frac{\psi_t-\psi}{C^{1-\sigma}} =& -\frac{\sigma+\varphi}{2}\hat{Y}_t^2 - \frac{\nu}{2}\hat{\pi}_t^2 - \frac{1}{2}\lambda(1-\lambda)(\sigma+\varphi)\frac{\sigma}{\varphi}\left(1-\zeta\frac{\sigma}{\sigma+\varphi}\right)\hat{q}_t^2\\ &-\frac{1+\varphi}{2}\zeta(1-\zeta)\left(\hat{L}_t - \hat{u}_t\right)^2\end{aligned} \tag{A.8}$$

We use (21) to replace $(\hat{L}_t - \hat{u}_t)$:

$$\begin{aligned}\frac{\psi_t-\psi}{C^{1-\sigma}} =& -\frac{\sigma+\varphi}{2}\hat{Y}_t^2 - \frac{\nu}{2}\hat{\pi}_t^2\\ &-\frac{1}{2}\lambda(1-\lambda)(\sigma+\varphi)\frac{\sigma}{\varphi}\left(1-\zeta\frac{\sigma}{\sigma+\varphi}\left(1-\frac{\lambda}{1-\lambda}\frac{\varphi}{1+\varphi}(1-\zeta)\right)\right)\hat{q}_t^2\end{aligned} \tag{A.9}$$

The welfare function can now be written as expressed in Proposition 5:

$$E_0\sum_{t=0}^{\infty}\beta^t\frac{\psi_t-\psi}{C^{1-\sigma}} = -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Psi_t + t.i.p. \tag{A.10}$$

with t.i.p. as terms independent of policy and where the per-period loss is given by

$$\Psi_t = (\sigma+\varphi)\hat{Y}_t^2 + \nu\hat{\pi}_t^2 + \lambda(1-\lambda)(\sigma+\varphi)\frac{\sigma}{\varphi}\Omega\hat{q}_t^2,$$

where $\Omega \equiv 1-\zeta\frac{\sigma}{\sigma+\varphi}\left(1-\frac{\lambda}{1-\lambda}\frac{\varphi}{1+\varphi}(1-\zeta)\right)$.

Appendix B: Model Summary

Table B.1 summarizes the log-linearized model.

Table B.1: Log-linearized THANK model

Description	Model equation
Households	
Consumption of S	$\hat{C}_t^S = (1-\zeta)(\hat{w}_t + \hat{L}_t^S) + \zeta \frac{1-\tau}{1-\lambda}(\hat{r}_t + \hat{u}_t) + \frac{1-\tau}{1-\lambda}\hat{\Pi}_{Y,t} = \frac{1-\lambda\chi}{1-\lambda}\hat{Y}_t$
Consumption of N	$\hat{C}_t^N = (1-\zeta)(\hat{w}_t + \hat{L}_t^N) + \zeta \frac{\tau}{\lambda}(\hat{r}_t + \hat{u}_t) + \frac{\tau}{\lambda}\hat{\Pi}_{Y,t} = \chi\hat{Y}_t$
Inequality	$\hat{q}_t = \hat{C}_t^S - \hat{C}_t^N = \frac{1-\chi}{1-\lambda}\hat{Y}_t$
	with $\chi \equiv 1 + \varphi\left(1 - \frac{\tau}{\lambda}\right)\kappa$
	and $\kappa \equiv \frac{1-\zeta\frac{1+\sigma+\varphi}{\sigma+\varphi}}{1-\zeta\frac{\sigma}{\sigma+\varphi}\left(1-\varphi\frac{\lambda}{1-\lambda}(1-\zeta)\left(1-\frac{\tau}{\lambda}\right)\right)}$
Aggregate Consumption	$\hat{C}_t = E_t\hat{C}_{t+1} - \lambda(\hat{q}_t - E_t\hat{q}_{t+1}) - (1-\alpha)E_t\hat{q}_{t+1}$
	$-\sigma^{-1}(\hat{R}_t - E_t\hat{\pi}_{t+1})$
Labor Supply	$\hat{W}_t = \varphi\hat{L}_t + \sigma\hat{C}_t$
Capital Utilization Services Supply	$\hat{r}_t = \varphi\hat{u}_t + \sigma\hat{C}_t^S$
Firms	
Labor Demand	$\hat{L}_t + \hat{w}_t = \hat{mc}_t + \hat{x}_t$
Capital Utilization Services Demand	$\hat{u}_t + \hat{r}_t = \hat{mc}_t + \hat{x}_t$
Phillips Curve	$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{\epsilon}{\upsilon}\hat{mc}_t + \epsilon_t^{mc}$
Marginal Costs	$\hat{mc}_t = (\sigma+\varphi)\hat{Y}_t + \sigma\lambda\zeta\hat{q}_t$
Aggregate Production	$\hat{Y}_t = \zeta\hat{u}_t + (1-\zeta)\hat{L}_t$
Profits	$\hat{\Pi}_{Y,t} = -\hat{mc}_t$
Goods Market Clearing	$\hat{Y}_t = \hat{C}_t$
Factor Markets Clearing	$\hat{L}_t = \frac{1}{1+\varphi}(\hat{mc}_t + \hat{x}_t - \sigma\hat{C}_t)$
	$\hat{u}_t = \frac{1}{1+\varphi}(\hat{mc}_t + \hat{x}_t - \sigma\hat{C}_t^S)$

Note: Variables with a hat describe log-deviations from their steady state (e.g., $\hat{C}_t = \ln C_t - \ln C$). In equilibrium, the labor share of total income is given by $wL/Y = 1-\zeta$ and the capital share of income by $ru/Y = \zeta(1-\lambda)^{-1}$.